An Introduction to Tire Modelling for Multibody Dynamics Simulation

SD 652
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Acknowledgement:


SAE Axis System

Aligning Torque ($M_z$)

Positive Camber Angle

Rolling Resistance Moment ($M_y$)

Overturning Moment ($M_x$)

Normal Force ($F_z$)

Tractive Force ($F_x$)
  Direction of Wheel Heading

Direction of Wheel Travel

Positive Slip Angle

Lateral Force ($F_y$)
ISO Axis System

- Spin axis
- Direction of wheel heading
- Direction of wheel travel
- Lateral force $F_y$
- Tractive force $F_x$
- $Y_{ISO}$
- $X_{ISO}$
Rolling Resistance ($M_y$)

Produced by hysteresis in tire tread and sidewall rubber

Normal force $F_z$ is integral of the distributed load

$$M_y = (F_z)(\delta x)$$
Braking Force \( (F_x) \)

\[
S = \frac{(V - wR)}{V}
\]

\(0 < S < 1\)
Braking Force \( (F_x) \)

\[
S = \frac{(V - wR)}{V}
\]

\( 0 < S < 1 \)

Longitudinal Stiffness, \( C_s \), is the slope of the \( F_x \) vs. \( S \) curve at \( S=0 \).
Driving Force ($F_x$)

$$S = \frac{(wR-V)}{wR}$$

$$0 < S < 1$$
Lateral Force ($F_y$) and Aligning Moment ($M_z$)

**SIDE VIEW**
- Pressure $p$
- Limit lateral stress $\mu p$
- Slipping starts
- Lateral stress
- Tyre contact patch
- Front
- Rear

**TOP VIEW**
- $M_z = F_y x_{pt}$
- Slipping starts
- Direction of wheel heading
- Direction of wheel travel
- Lateral stress
- Pneumatic trail

Aligning moment due to slip angle
- Lateral force
- Pneumatic trail
- Direction of travel

$\alpha$
Lateral Force ($F_y$) and Aligning Moment ($M_z$)

Cornering Stiffness, $C_\alpha$, is the slope of the $F_y$ vs. $\alpha$ curve at $\alpha=0$
Effect of Camber Angle ($\gamma$) on Lateral Force ($F_y$)
Combined Slip ($F_x$ AND $F_y$)
Overturning Moment ($M_x$)
Characterizing a Pneumatic Tire: Physical Testing

\[ F_z = \text{normal force} \]

\[
\begin{align*}
F_x & = F_x(F_z, S, \alpha, \gamma) \\
F_y & = F_y(F_z, S, \alpha, \gamma) \\
M_x & = M_x(F_z, S, \alpha, \gamma) \\
M_y & = M_y(F_z, S, \alpha, \gamma) \\
M_z & = M_z(F_z, S, \alpha, \gamma)
\end{align*}
\]
Characterizing a Pneumatic Tire: Physical Testing
Characterizing a Pneumatic Tire: Physical Testing
Consider how $F_x$ varies with $F_z$, $S$, $\alpha$, $\gamma$: $20^4 = 160,000$ data points

$F_z = \text{normal force}$

$F_x = F_x(F_z, S, \alpha, \gamma)$

$F_y = F_y(F_z, S, \alpha, \gamma)$

$M_x = M_x(F_z, S, \alpha, \gamma)$

$M_y = M_y(F_z, S, \alpha, \gamma)$

$M_z = M_z(F_z, S, \alpha, \gamma)$

$x5 = 800,000$ data points
**Tire Models:**

Mathematical Functions to Fit Measured Data

**Fiala:** 6 parameters needed to describe a tire

\[
\begin{align*}
F_x &= F_x(F_z, S, \alpha, \gamma) \\
F_y &= F_y(F_z, S, \alpha, \gamma) \\
M_x &= M_x(F_z, S, \alpha, \gamma) \\
M_y &= M_y(F_z, S, \alpha, \gamma) \\
M_z &= M_z(F_z, S, \alpha, \gamma)
\end{align*}
\]

**Pacejka 2002 : 117 parameters needed**

\[
\begin{align*}
F_x &= F_x(S) \\
F_y &= F_y(\alpha) \\
M_x &= 0 \\
M_y &= M_y(F_z) \\
M_z &= M_z(\alpha)
\end{align*}
\]
How Tire Forces are Included In Multibody Vehicle Model

1. Define a point where tire forces and moments will act on the multibody model

\[
\vec{F}_C = \vec{F}_P \\
\vec{M}_C = \vec{M}_P + \vec{R}_{P/C} \times \vec{F}_P
\]
2. Determine an expression for the vertical tire force, $F_z$, which is required as an input to the tire model.

$$F_z = \max \left( k_z \delta_z + c_z \dot{\delta}_z, 0 \right)$$
How Tire Forces are Included In Multibody Vehicle Model

3. Establish vector directions for longitudinal and lateral components of tire force.

\[
\hat{u}_x = \frac{\hat{u}_{RevAxis} \times \hat{u}_z}{|\hat{u}_{RevAxis} \times \hat{u}_z|}
\]

\[
\hat{u}_y = \hat{u}_z \times \hat{u}_x
\]
How Tire Forces are Included In Multibody Vehicle Model

4. Determine kinematic inputs to tire model \((S, \gamma, \alpha)\)
5. Use a tire model to calculate \(F_x, F_y, M_x, M_y, M_z\)
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