An Introduction to Tire Modelling for Multibody Dynamics Simulation

SD 652

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University of Waterloo

Acknowledgement:

Kevin Morency, *Automatic Generation of Real-Time Simulation Code for Vehicle Dynamics using Linear Graph Theory and Symbolic Computing*, MASc Thesis, 2007

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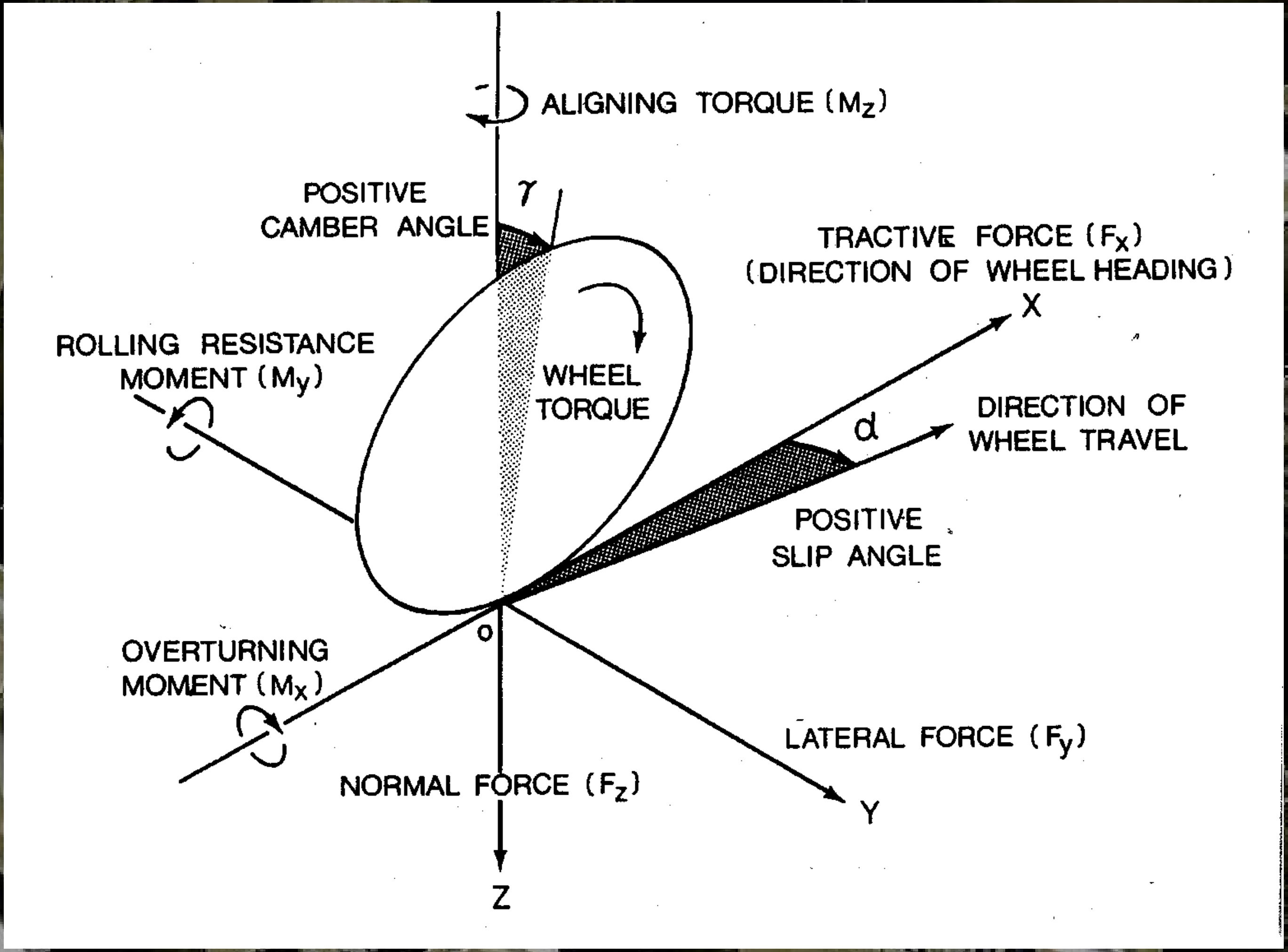
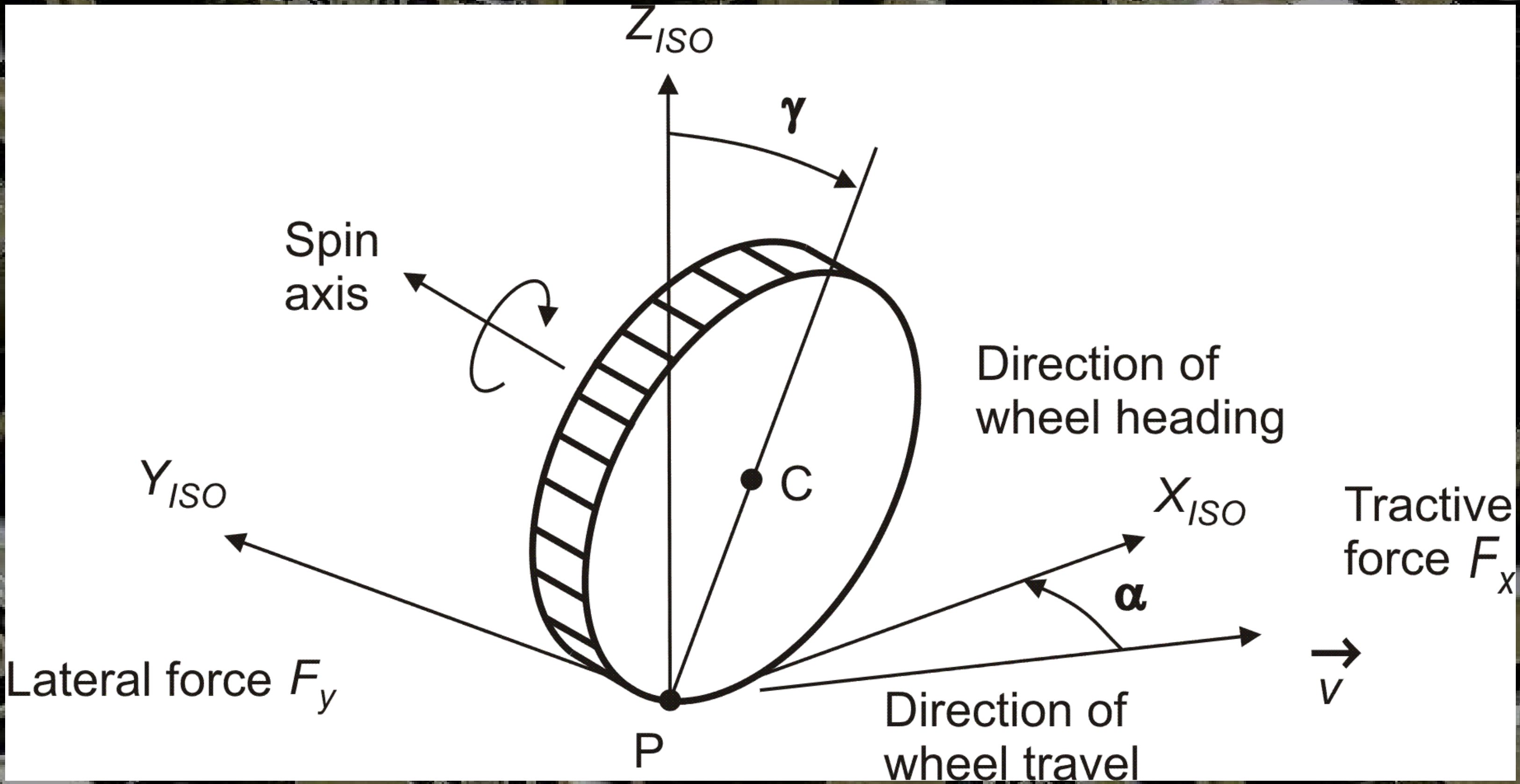
Matthew Van Gennip, *Vehicle Dynamic Modelling and Parameter Identification for an Autonomous Vehicle*, MASc Thesis, 2018

SAE Axis System

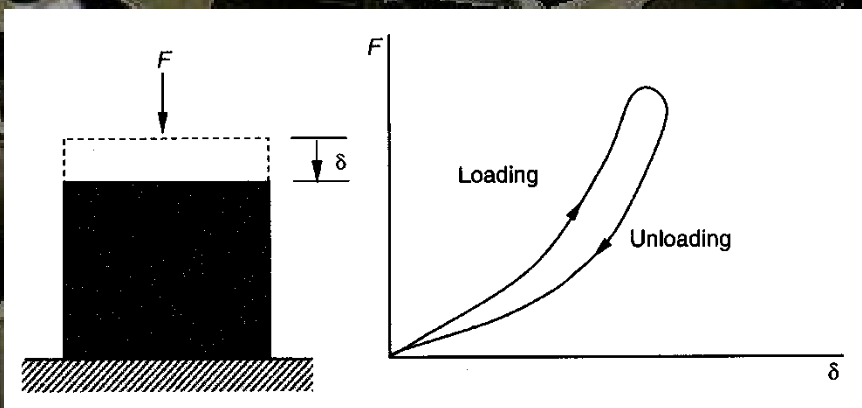


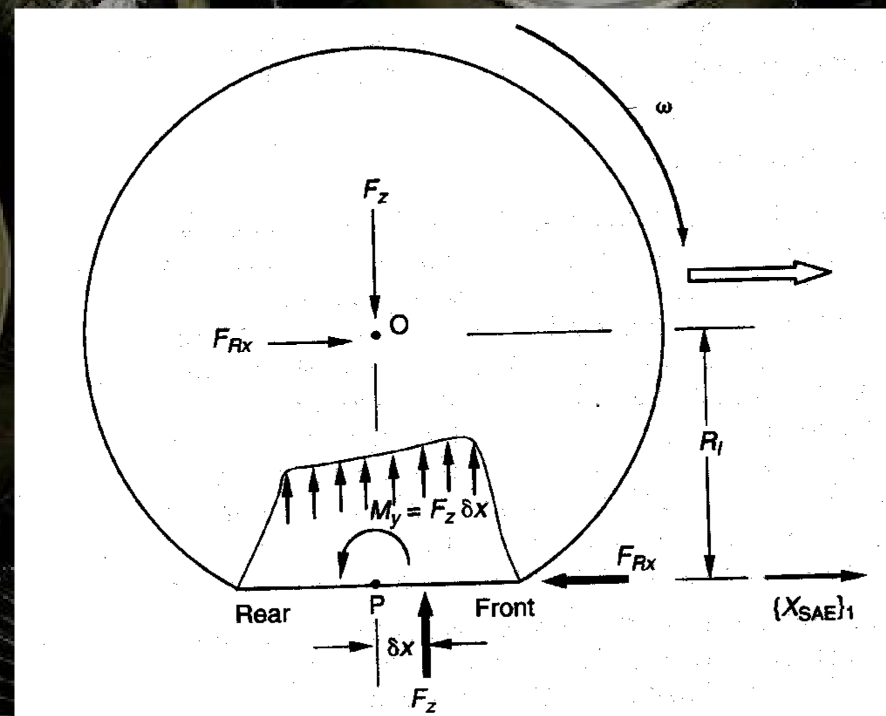
ISO Axis System



Rolling Resistance (M_y)

Produced by hysteresis in tire tread and sidewall rubber





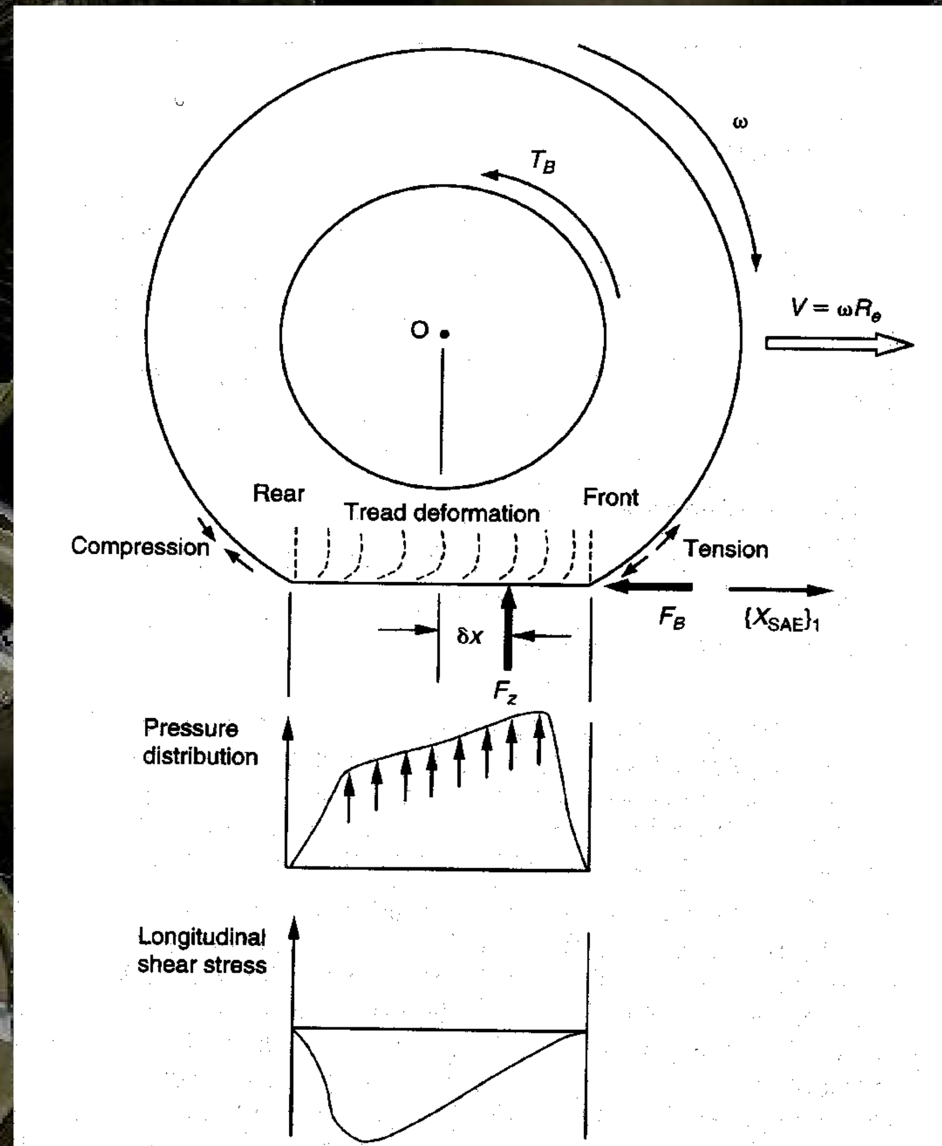
Normal force Fz is integral of the distributed load

$$M_y = (Fz)(\delta x)$$

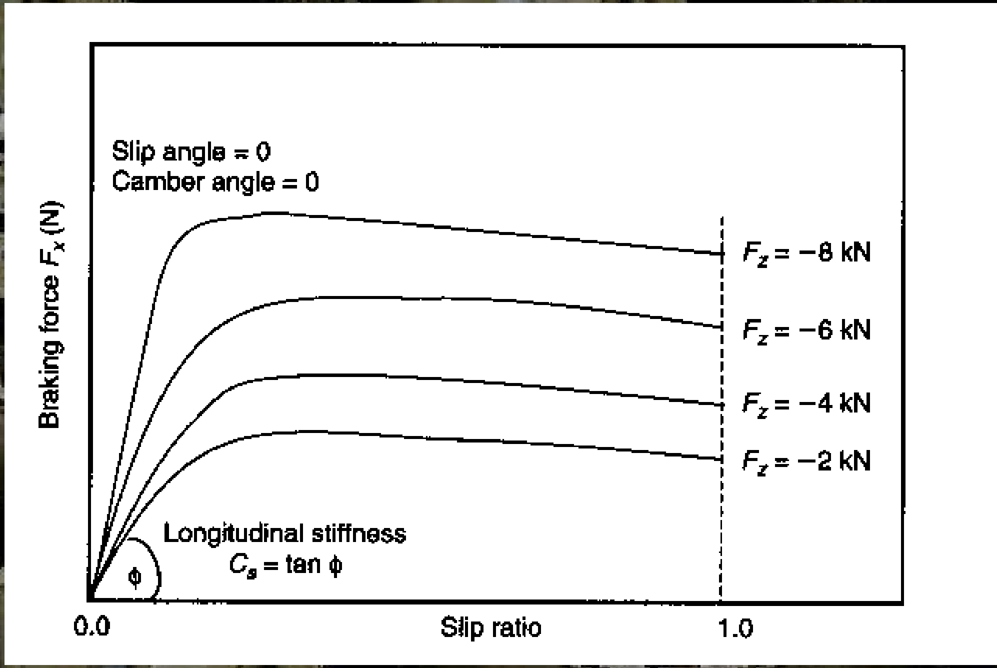
Braking Force (F_x)

$$S = (V - wR) / V$$

$$0 < S < 1$$



Braking Force (F_x)



$S = (V - wR) / V$

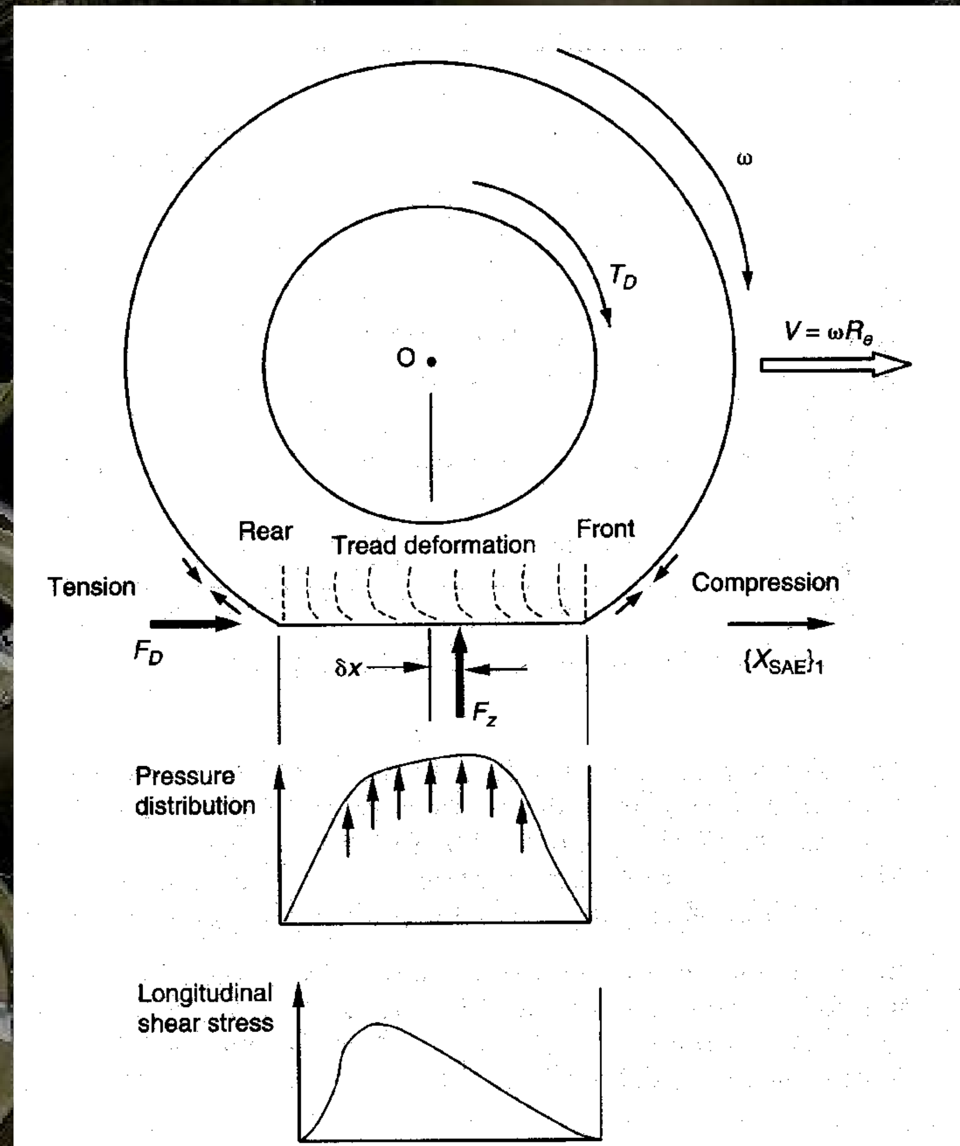
$0 < S < 1$

Longitudinal Stiffness, C_S, is the slope of the F_x vs. S curve at S=0

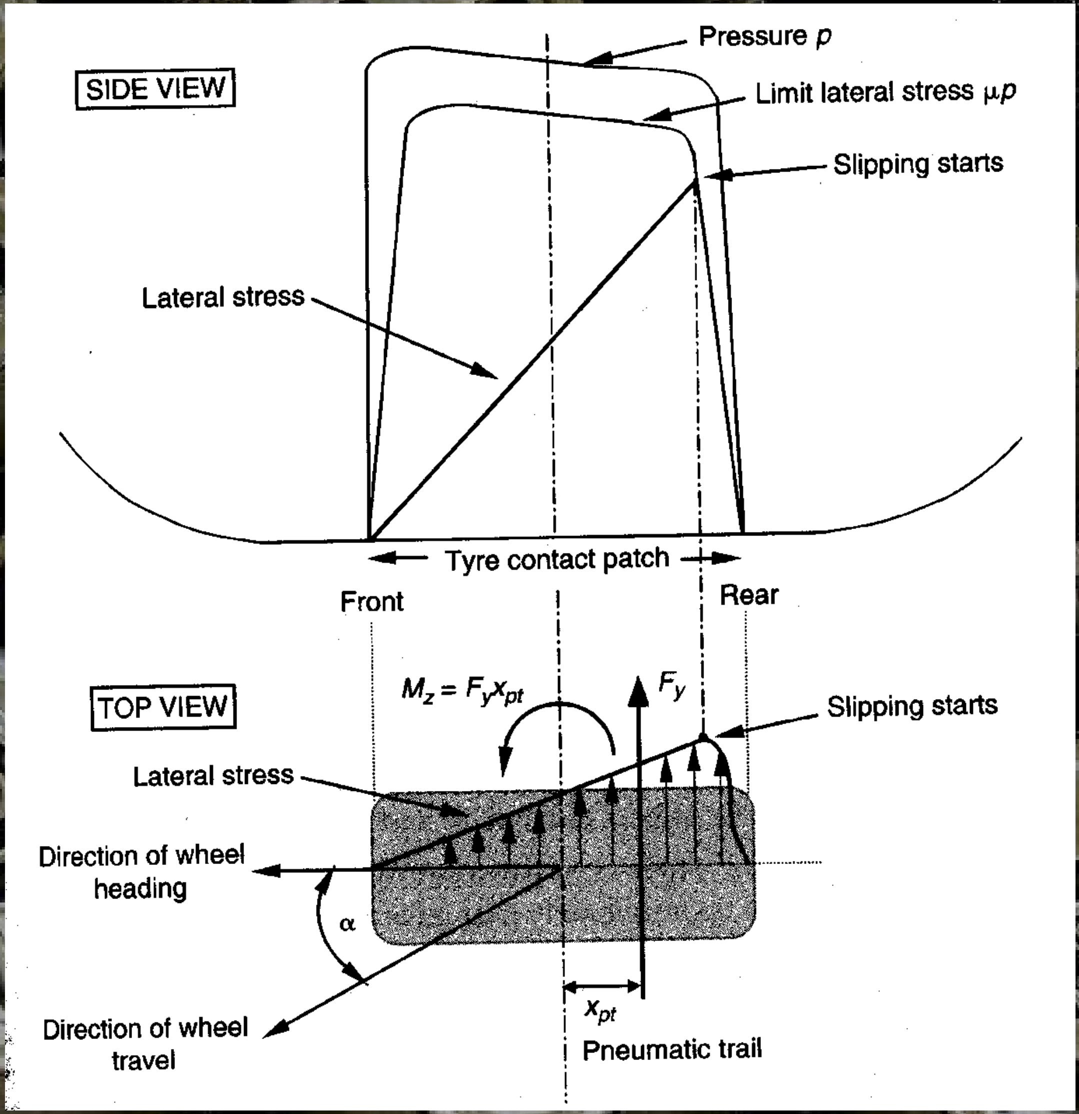
Driving Force (F_x)

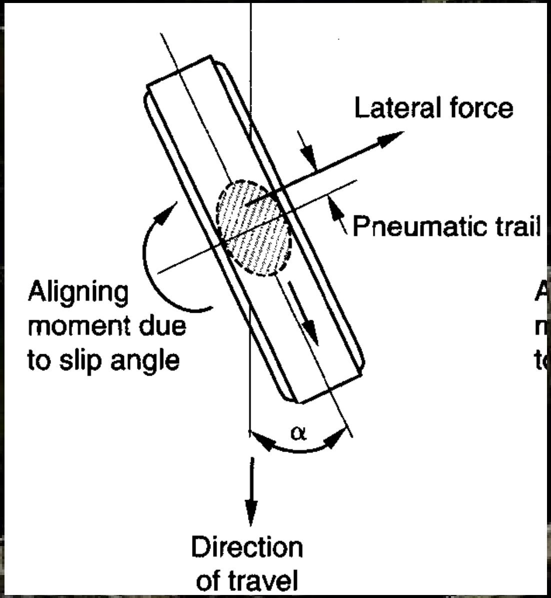
$S = (wR - V) / wR$

$0 < S < 1$



Lateral Force (F_y) and Aligning Moment (M_z)





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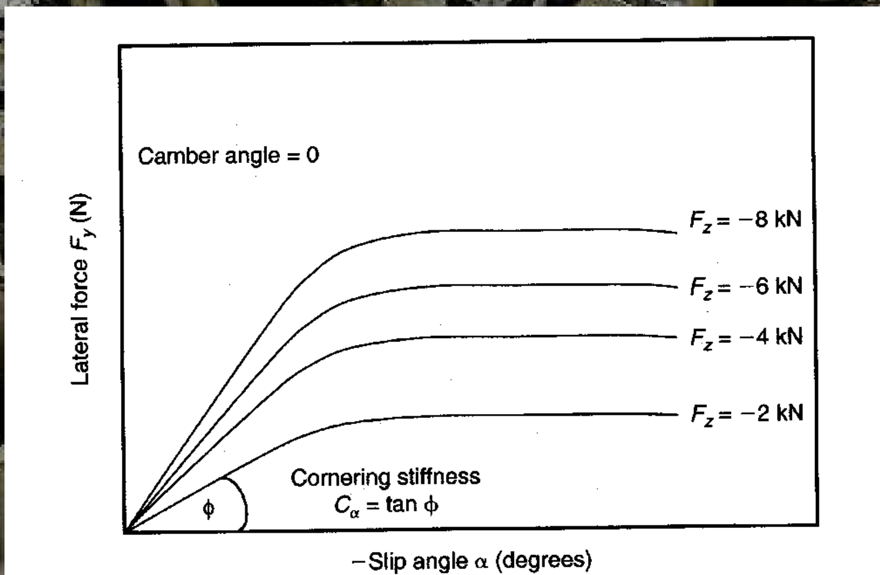
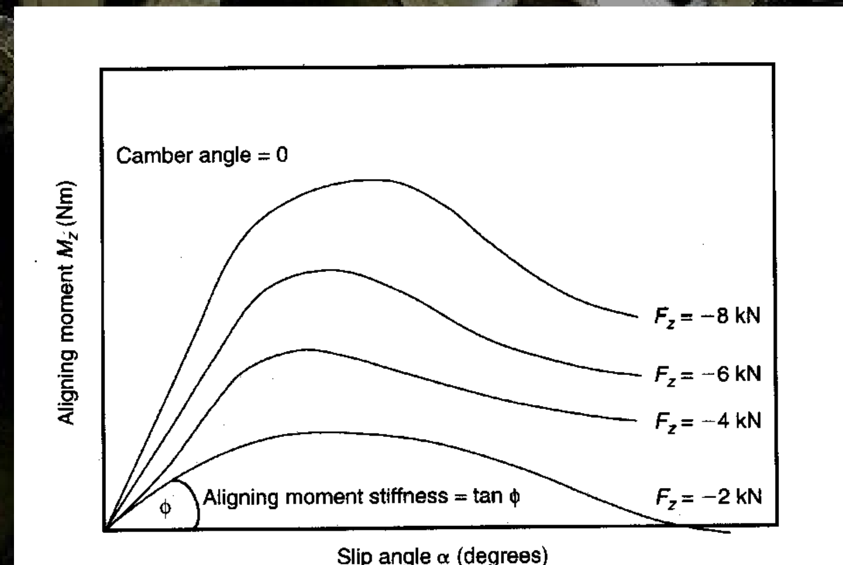
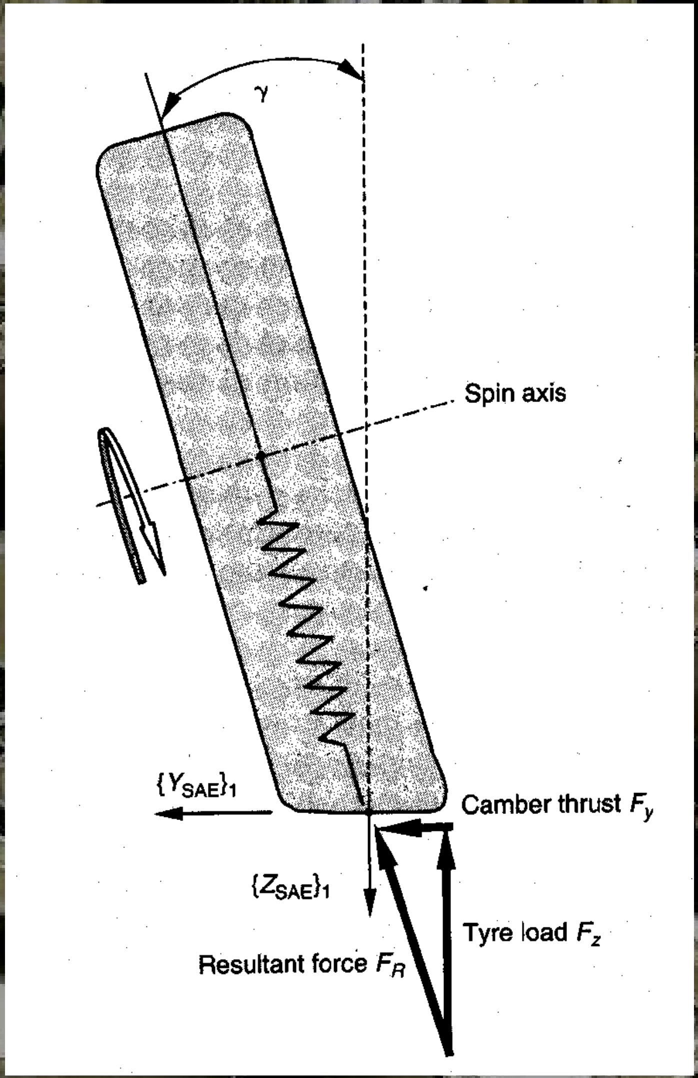
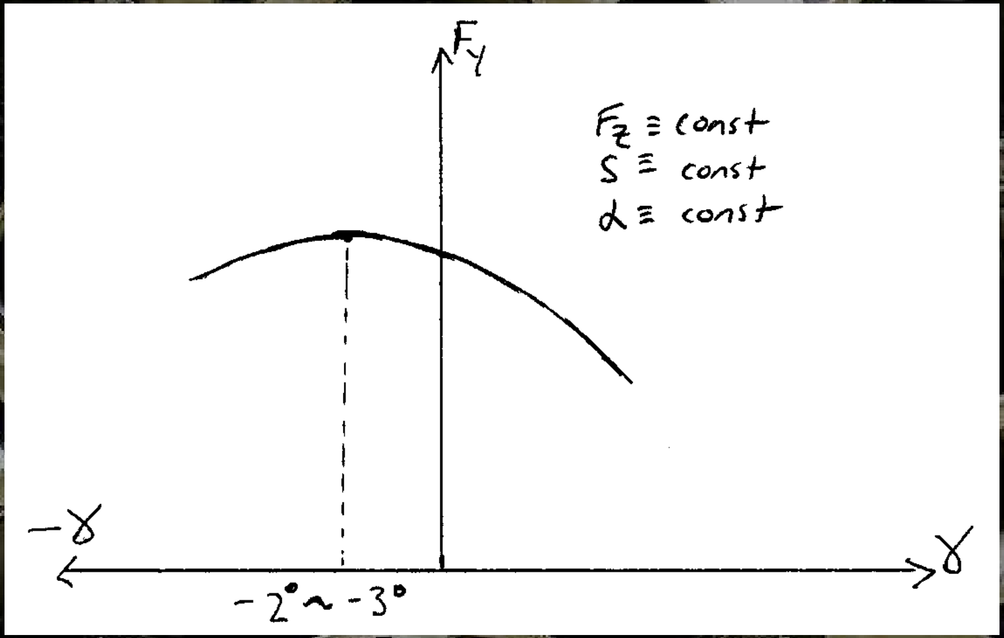




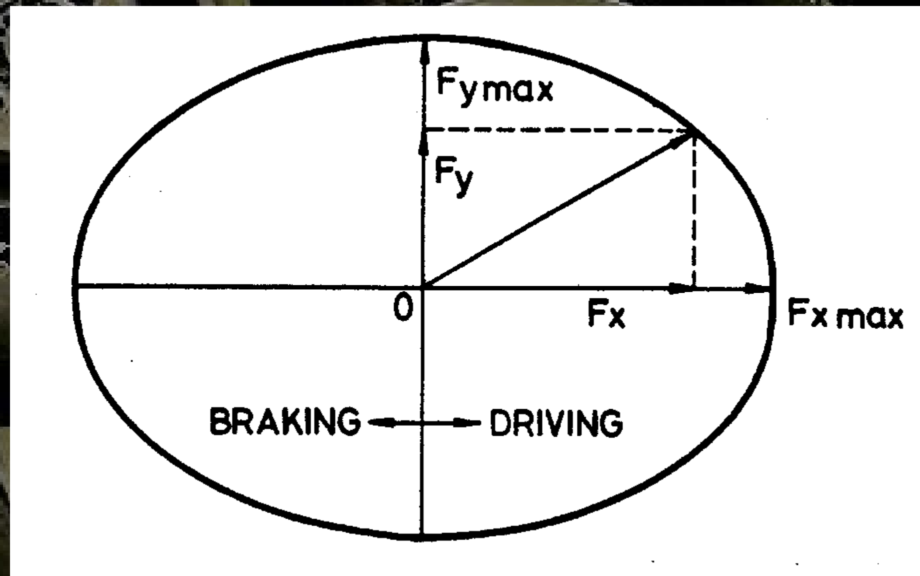
Cornering Stiffness, C_α, is the slope of the F_y vs. α curve at $\alpha=0$

Effect of Camber Angle (γ) on Lateral Force (F_y)

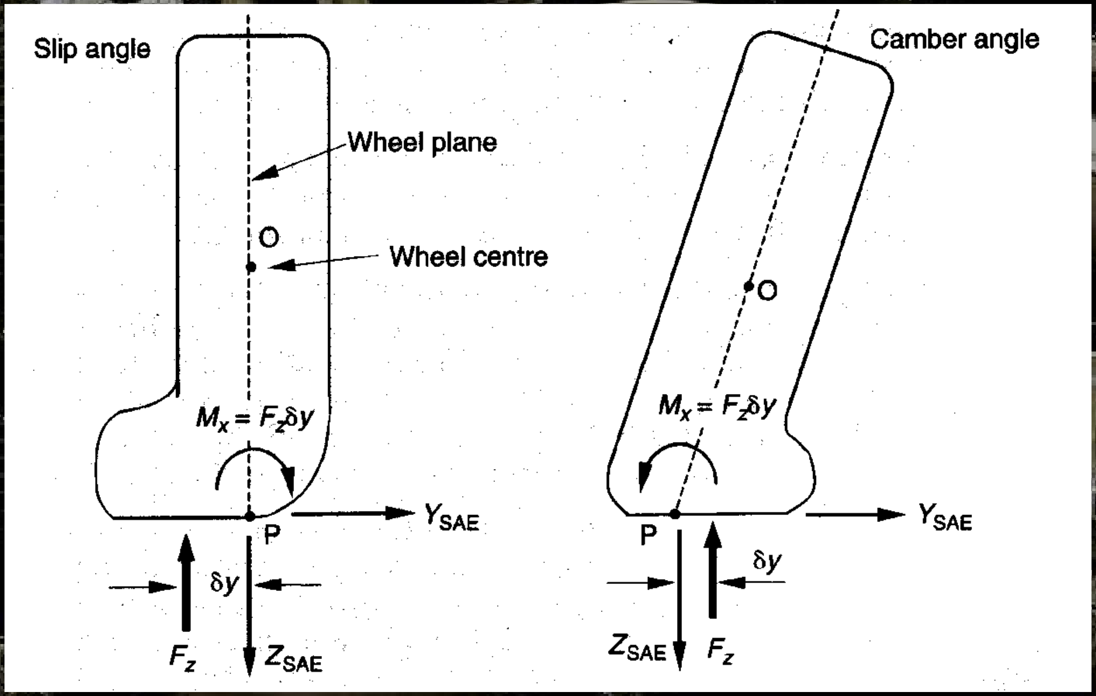




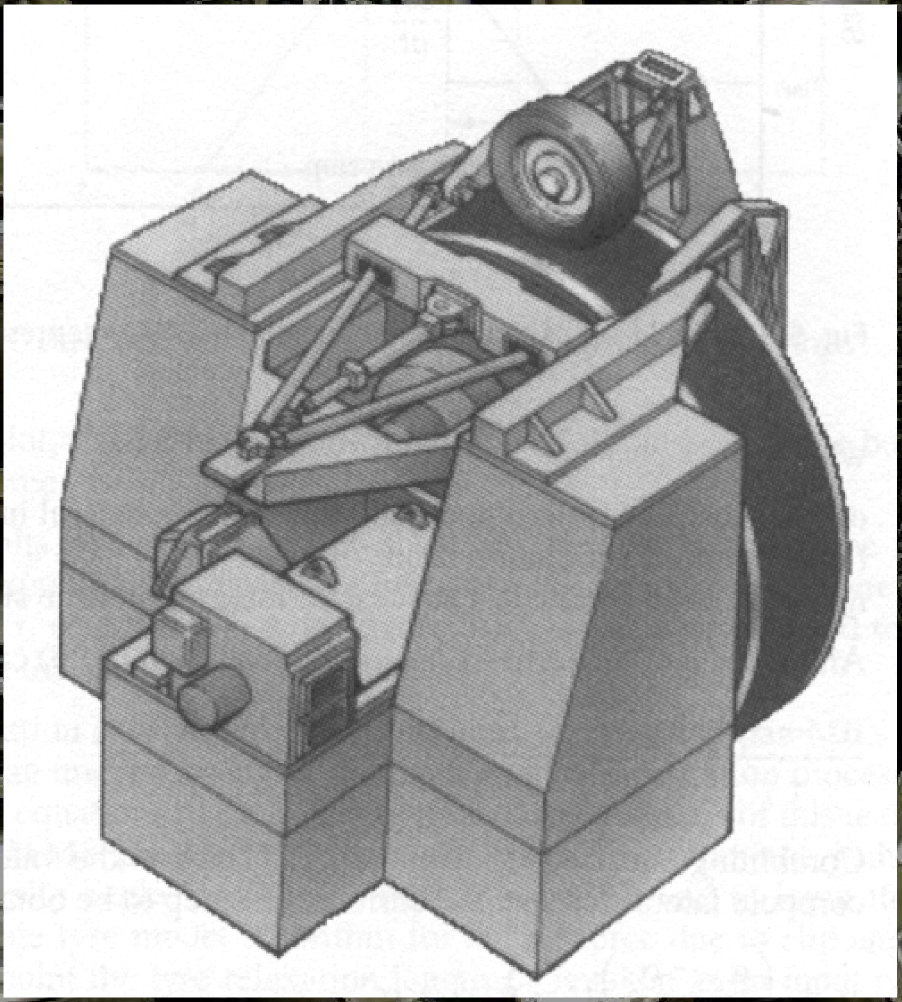
Combined Slip (F_x AND F_y)



Overturning Moment (M_x)



Characterizing a Pneumatic Tire: Physical Testing



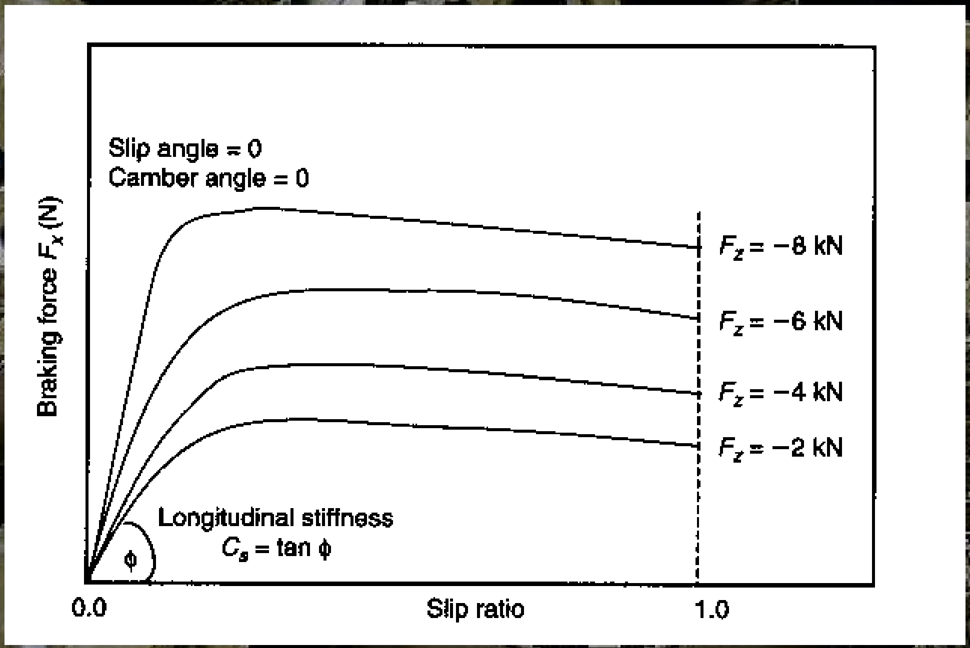
$$Fz = normal\,force$$

$$
\begin{aligned}
Fx &= Fx(Fz, S, \alpha, \gamma) \\
Fy &= Fy(Fz, S, \alpha, \gamma) \\
Mx &= Mx(Fz, S, \alpha, \gamma) \\
My &= My(Fz, S, \alpha, \gamma) \\
Mz &= Mz(Fz, S, \alpha, \gamma)
\end{aligned}
$$

Characterizing a Pneumatic Tire: Physical Testing

Characterizing a Pneumatic Tire: Physical Testing

Data From Physical Tests



$$Fz = normal force$$

$$\begin{aligned} Fx &= Fx(Fz, S, \alpha, \gamma) \\ Fy &= Fy(Fz, S, \alpha, \gamma) \\ Mx &= Mx(Fz, S, \alpha, \gamma) \\ My &= My(Fz, S, \alpha, \gamma) \\ Mz &= Mz(Fz, S, \alpha, \gamma) \end{aligned}$$

Consider how Fx varies with Fz, S, α, γ : 20^4 =

160 000 data points

x5 = 800 000 data points

Tire Models:

Mathematical Functions to Fit Measured Data

Fiala: 6 parameters needed to describe a tire

$$\begin{aligned} Fx &= Fx(Fz, S, \alpha, \gamma) \\ Fy &= Fy(Fz, S, \alpha, \gamma) \\ Mx &= Mx(Fz, S, \alpha, \gamma) \\ My &= My(Fz, S, \alpha, \gamma) \\ Mz &= Mz(Fz, S, \alpha, \gamma) \end{aligned}$$

→

$$\begin{aligned} Fx &= Fx(S) \\ Fy &= Fy(\alpha) \\ Mx &= 0 \\ My &= My(Fz) \\ Mz &= Mz(\alpha) \end{aligned}$$

Pacejka 2002 : 117 parameters needed

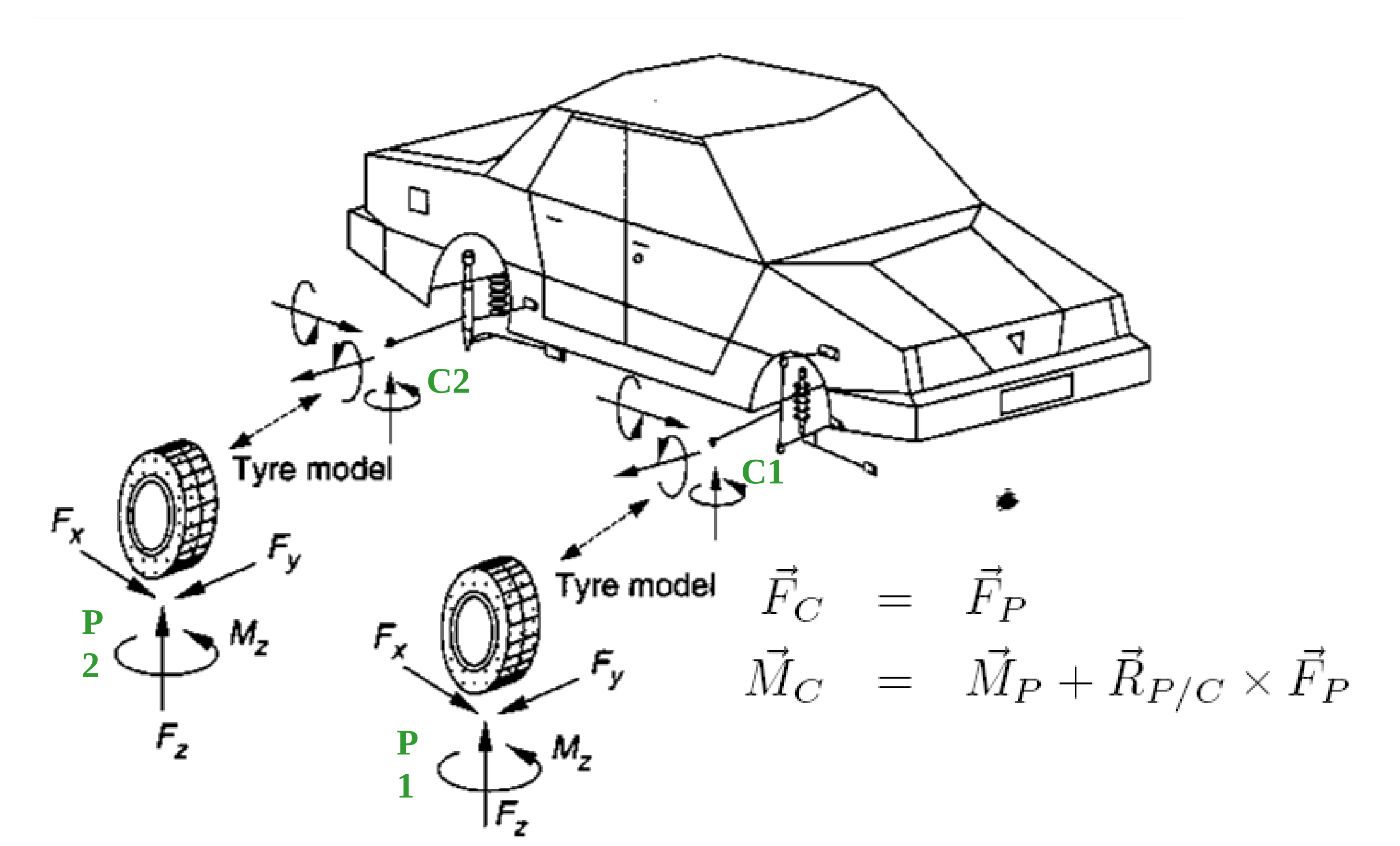
$$\begin{aligned} Fx &= Fx(Fz, S, \alpha, \gamma) \\ Fy &= Fy(Fz, S, \alpha, \gamma) \\ Mx &= Mx(Fz, S, \alpha, \gamma) \\ My &= My(Fz, S, \alpha, \gamma) \\ Mz &= Mz(Fz, S, \alpha, \gamma) \end{aligned}$$

→

$$\begin{aligned} Fx &= Fx(Fz, S, \alpha, \gamma) \\ Fy &= Fy(Fz, S, \alpha, \gamma) \\ Mx &= Mx(Fz, S, \alpha, \gamma) \\ My &= My(Fz, S, \alpha, \gamma) \\ Mz &= Mz(Fz, S, \alpha, \gamma) \end{aligned}$$

How Tire Forces are Included In Multibody Vehicle Model

1. Define a point where tire forces and moments will act on the multibody model

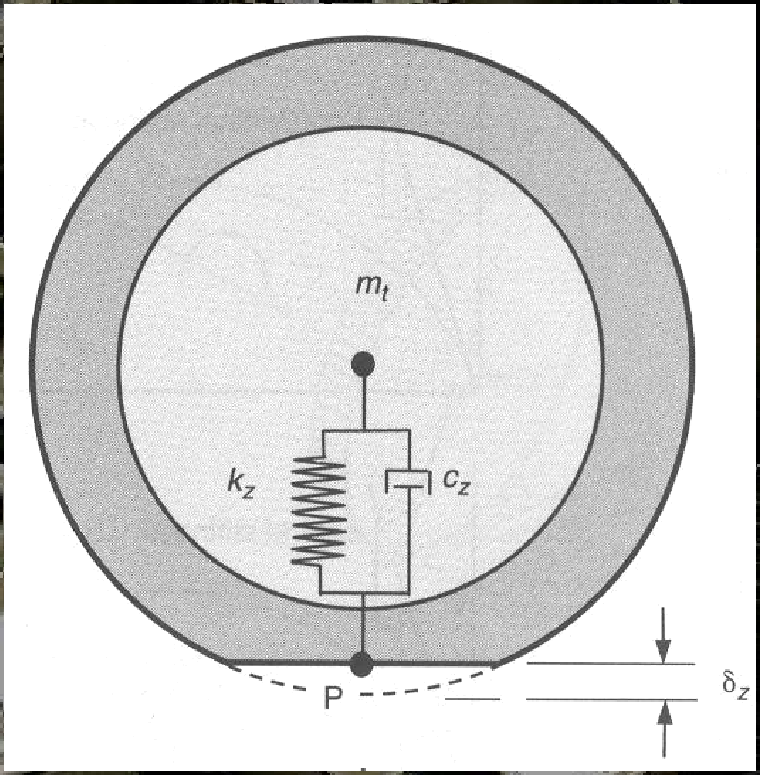


$$\vec{F}_C = \vec{F}_P$$

$$\vec{M}_C = \vec{M}_P + \vec{R}_{P/C} \times \vec{F}_P$$

How Tire Forces are Included In Multibody Vehicle Model

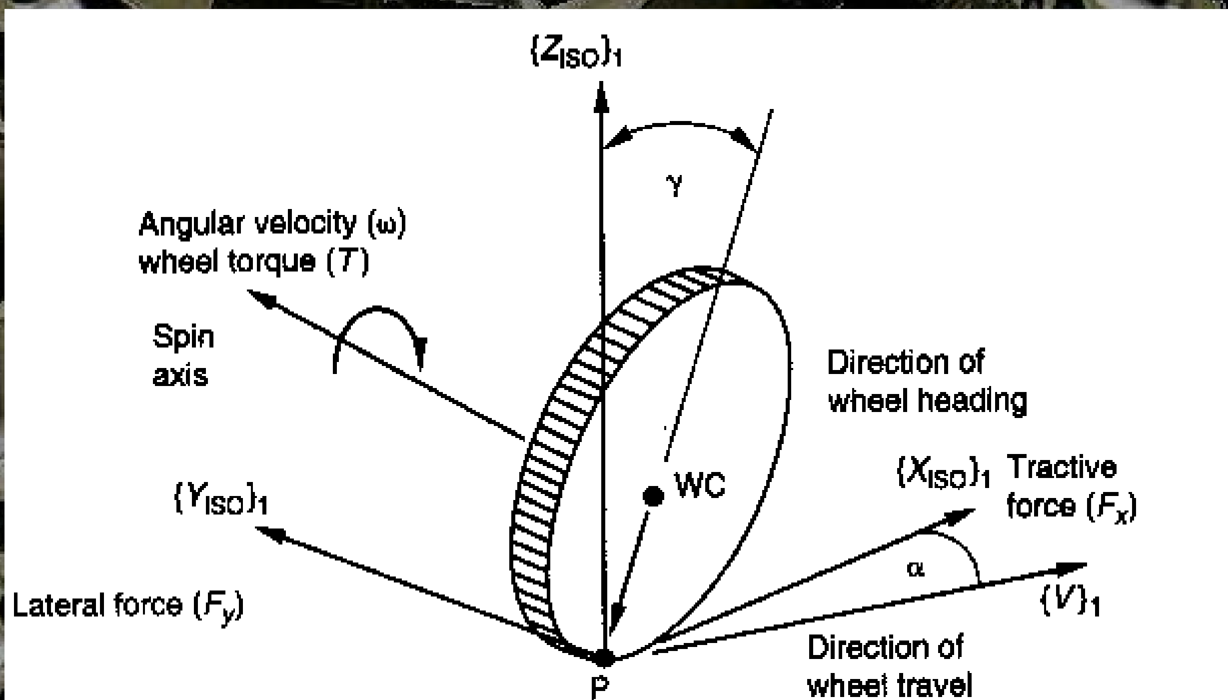
2. Determine an expression for the vertical tire force, Fz, which is required as an input to the tire model.



$$Fz = max\left(k_z \delta_z + c_z \dot{\delta_z}, 0\right)$$

How Tire Forces are Included In Multibody Vehicle Model

3. Establish vector directions for longitudinal and lateral components of tire force.

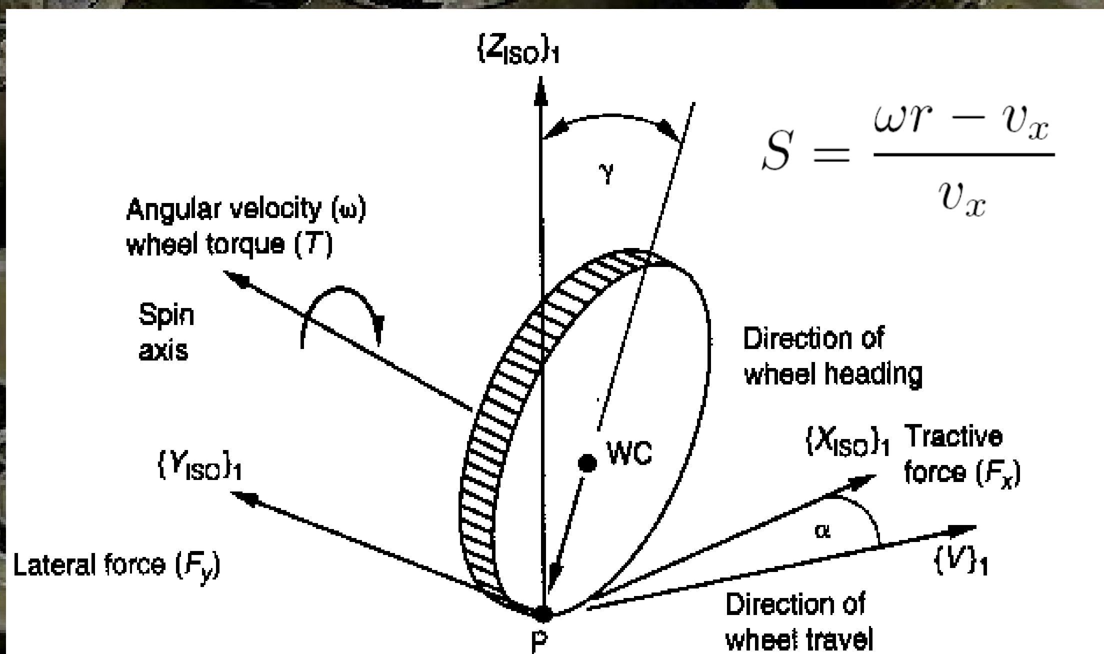


$$\hat{u}_x = \frac{\hat{u}_{RevAxis} \times \hat{u}_z}{\left|\hat{u}_{RevAxis} \times \hat{u}_z\right|}$$

$$\hat{u}_y = \hat{u}_z \times \hat{u}_x$$

How Tire Forces are Included In Multibody Vehicle Model

4. Determine kinematic inputs to tire model (S, γ, α)
5. Use a tire model to calculate Fx, Fy, Mx, My, Mz



$$S = \frac{\omega r - v_x}{v_x}$$

$$\begin{aligned} Fx &= Fx(Fz, S, \alpha, \gamma) \\ Fy &= Fy(Fz, S, \alpha, \gamma) \\ Mx &= Mx(Fz, S, \alpha, \gamma) \\ My &= My(Fz, S, \alpha, \gamma) \\ Mz &= Mz(Fz, S, \alpha, \gamma) \end{aligned}$$

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