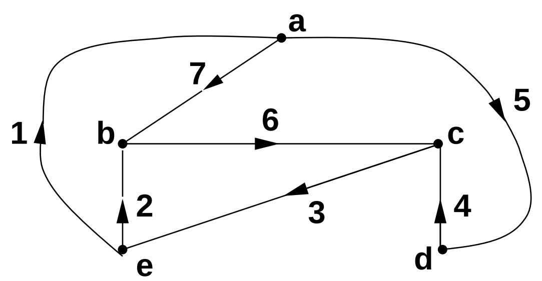
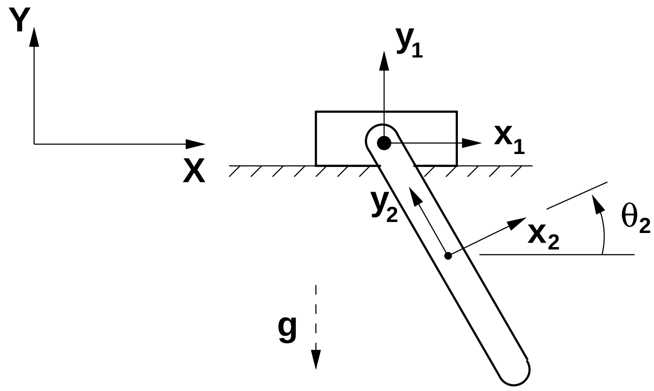
Systems Design 652

Assignment 4 — Due 27 March 2012

[1]: Derive *independently* the fundamental circuit matrix, fundamental cutset matrix, and incidence matrix (in that order) for the graph below. Use edges 1, 2, 3, and 4 as the tree. Check your answer by showing that $\mathbf{A_c} = -\mathbf{B_b}^T$, and that the cutset matrix can be obtained from the incidence matrix using elementary row operations. Sketch the cutsets on a figure of the graph.



[2]: A pendulum of length $2l$ is pinned to a block that slides along a horizontal axis. The block and pendulum have masses m_1 and m_2, respectively, and centroidal moments of inertia of I_1 and I_2. Gravity g acts in the $-Y$ direction.



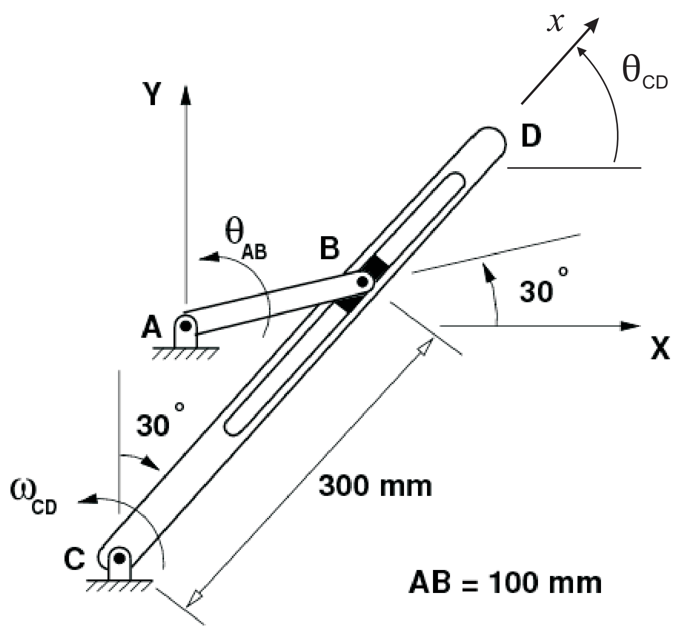
Draw a linear graph of the system, and select separate translational and rotational trees that will lead to a minimum number of motion equations in x_1 and θ_2. Use a graph-theoretic approach to derive these dynamic equations of motion by hand, expressed in standard matrix form, and explain their physical significance.

[3]: The "quick-return" mechanism consists of a crank AB, slider block B, and slotted link CD. The crank is being driven with a constant angular speed given by the function $\theta_{AB} = \pi/6 + 2\pi t$ (rad), where the time t is in seconds. The center of mass of the crank is 40 mm from A, while the mass center of link CD is 100 mm from C.

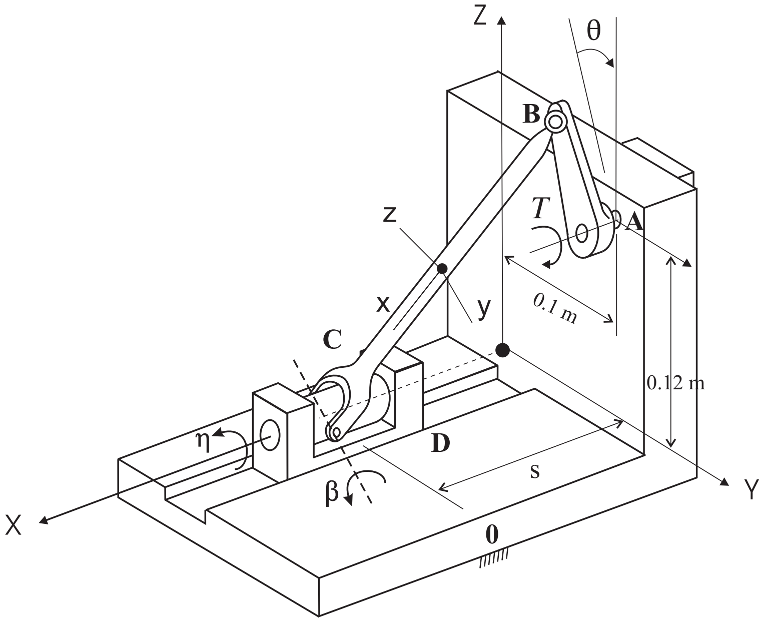
(a). Draw a linear graph for the mechanism and select separate translational and rotational trees that will lead to a minimal number of kinematic equations. In comparison, how many coordinates would ADAMS use? How many kinematic equations would result from a joint coordinate formulation?

(b). Use a graph-theoretic approach to formulate the governing kinematic equations at the position and velocity level. For the latter, do not simply differentiate the position equations; instead, use the circuit equations in terms of velocity vectors.

(c). Solve these equations (analytically or numerically) and plot the angular displacement θ_{CD} and velocity ω_{CD} of the slotted link CD versus time, for one full revolution of the crank. What is the maximum speed of the slider B relative to CD, and when does it occur? Explain.



[4]: The spatial slider-crank mechanism [Haug, 1989, pp.396-401] consists of a crank of length 0.08 m, a connecting rod of length 0.24 m, and a sliding block. The crank, connected to the ground by revolute joint A, is being driven at a constant angular velocity such that $\theta = 2\pi\, t$ rad. There is a spherical joint at B and a universal joint at C, with universal joint angles η and β defined in the second figure. The block is constrained to the ground by a prismatic joint D with sliding displacement s.



Draw a single linear graph for this system, and select a single tree that results in the branch coordinates $\mathbf{q} = [s, \theta, \eta, \beta]^T$. Use the projected circuit equations to derive the kinematic constraint equations at the position level. Solve these equations for the position s of the slider versus time, either analytically or numerically, and plot it for one cycle of the crank rotation.