Control of a
Flexible Link Robotic Manipulator
in Zero Gravity Conditions

by

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Chapter 1

Introduction

Traditional robots are constructed very massively to make them precise and stiff. The arms of these robots can be considered rigid, which allows a simple control of the joints. The drawback of their heavy construction is, that the robots need very powerful actuators and their operating speed is strongly limited by their own inertia.

Lightweight robots are developed to overcome these drawbacks and allow high speed movements with the same or even better precision. These are commonly used in space applications, because the take-off weight of space shuttles is strongly limited. These robots, like the Canadarm on International Space Station have to carry huge payloads compared to their own weight. In order to control the end-effector of such a robot, the complex dynamics of the joints and the arms have to be taken into account.

Motivated by the fascination of a zero gravity environment, this project dealt with several aspects of space manipulator control. Since servo actuators with harmonic drive gearing are commonly used for space manipulators, the major part of the project investigated their behavior and provided solutions to increase their performance. With an increased actuator performance, which is crucial for a good overall behavior of the manipulator, a couple of different control approaches were applied.

The experiments were performed on the Watflex facility at the University of Waterloo. Watflex simulates a space like environment for 2-D applications.
Figure 1.1: The 2DOF manipulator of the Watflex facility.

This report is split in four chapters:

Chapter 2 provides details about the structure of the Watflex facility.

Chapter 3 briefly presents the preparations done in order to perform the desired experiments.

Chapter 4 describes the investigation of friction in the robot actuators. It provides background information about friction, different friction models and their utilization for friction compensation. Experiments show the application of friction compensation on the Watflex actuators, and the results are presented.

Chapter 5 introduces two different control approaches and describes experiments. Due to time constraints, the passivity-based controller is limited to the initial implementation and the observer-based controller is limited to independent joint control.

Chapter 6 summarizes the results and gives ideas for further research.
Chapter 2

System description

The Waterloo Flexible Link Experimental facility, called Watflex, has been developed since 1997 by the Systems Design Engineering Department at the University of Waterloo. It is a two degree of freedom (2DOF) robotic manipulator on a table perpendicular to gravity. Together with air bearings mounted on each joint and the end-effector, this system mimics a space environment constrained to planar motion.

This Chapter briefly describes the main sub-systems of Watflex and introduces the sensors used for this research project. A more detailed system description can be found in [2] and [13].

2.1 Mechanical structure

The mechanical system is shown in Figure 2.1. The support structure consists of an aluminum table, a metal sheet, a melamine particle board and the glass surface. The table is designed to assure minimal deflections due to varying load locations and it has the capability to level the glass surface. After the levelling procedure, described in [2], had been done the maximum difference over the entire surface was 1mm.

Figure 2.2 shows the dimensions of the manipulator in mm. The two links are aluminum beams with cross-sections of 6.73mm × 38.4mm for Link 1 and 4.8mm × 38.2mm for Link
2.5m Instruments

Supporting Table (Pentium-100 PC)

Rate Gyro

Pneumatic System (100psi Compressed Air)

Motor Amplifiers (ElectroMate 30A20-AC)

Data Acquisition and Signal Conditioning (Intelligent Instruments)

Digital Frame Grabber and Digital Module (Matrox)

Figure 2.1: Overview of Watflex (Picture from [13]).
2. The thickness of the beams is so chosen, that the maximum torque of the motors cannot deform the beams plastically.

![Diagram of the manipulator dimensions](image)

Figure 2.2: Dimensions of the manipulator (in \textit{mm}).

The two actuators (see Paragraph 2.2) and the end-effector of the manipulator are equipped with air bearings to assure frictionless floating on the glass surface. The air for the bearings is supplied with a pressure of 6\textit{bar} to force a floating height of at least 0.6\textit{mm}. This floating height prevents the manipulator from contacting the surface during fast movements. The air supply for the shoulder motor can be switched between pressure and vacuum for both, complete free floating and fixed base experiments. In this research project, only the second case is used. Since the air bearing of the shoulder joint needs some further changes to assure proper contact to the glass surface in the vacuum mode, the bearing was additionally fixed with four small pads (1cm$^2$) of double-sided duct tape for all experiments done.

### 2.2 Actuators

Each joint of the manipulator is a DC-brush permanent magnet servomotor with 50 : 1 harmonic drive gearing$^{12}$. Currently, harmonic drive gearing is commonly used in space applications due to its high torque-to-mass ratio, high reduction ratio, compactness, and virtually zero backlash. The working principle is shown in Figure 2.3. The high reduction

---

$^1$Shoulder: HD Systems RFS-32-6030-TE124AL-SP

$^2$Elbow: HD Systems RFS-25-6018-TE124AL-SP
ratio and the very low backlash is achieved by using an elliptical wave generator to mate a flexspline to a circular spline. A drawback of this type of gearing is the high internal friction compared to other types of gearing, e.g., toothed gears. See Chapter 4 for the detailed investigation of the frictional behavior.

Linear pulse-width modulated (PWM) amplifiers are converting a control signal to the necessary input voltage for the actuators. These amplifiers are driven in current mode, i.e. the output current is proportional to the input voltage. The determination of the conversion factor between input voltage and output current is described in Section 3.1. The knowledge of the input currents for the motors allows calculation of a motor torque using the torque constants provided by the manufacturer.

### 2.3 Sensors

The Watflex facility is equipped with a variety of sensors to determine relative and absolute motion of the robot. The sensors that were used for this project are described in the sections below.
2.3.1 Optical encoders

To determine the angle of rotation of the joints, optical encoders with a physical resolution of $1024 \frac{\text{pulses}}{\text{revolution}}$ are integrated in the actuators. Since they are connected to the shaft of the DC-motor, the resolution for output angle measurements is multiplied by the gear ratio of 50 : 1. Additionally, a quadrature decoder can be used get a resolution multiplied by four. This leads to an overall resolution of $204800 \frac{\text{pulses}}{\text{revolution}}$, which is equivalent to angular quantization with $0.001758^\circ$ steps.

These encoders work incrementally, i.e. only the relative angle since the last reset of the counters is measured.

The angular velocity can be derived easily by differentiating the angular measurements.

2.3.2 Strain gages

For many methods to control a flexible link, the dynamic measurement of deflection and angle of the endpoint is necessary. Strain gages can be used to achieve this requirement. Since a flexible link performs not only at its fundamental mode but also at its higher modes, one strain measurement along the link cannot provide accurate dynamic results.

A common method to estimate the endpoint position and the endpoint angle is an approximation of the dynamic beam shape by an $n^{th}$-order polynomial. The number of necessary strain measurements along the link increases with $n$, because for each coefficient of the polynomial is at least one strain measurement or one boundary condition required. The influence of $n$ and the strain gage locations on the accuracy of the estimated endpoint position can be found in [10]. According to this publication and initial qualitative measurements, the location for the strain measurements should be equally spread along the link. To fulfil this requirement, new strain gages were mounted on the links and calibrated during the project, see Section 3.3.

Strain measurements can be done in several configurations with one, two, or four strain gages. Due to their small relative change in resistance during a measurement, the strain gages are usually wired as a Wheatstone-bridge, and the output voltage is measured by a
differential amplifier. Full bridges with four strain gages are preferred for bending measurements because they are self-compensated for temperature changes and torsional forces. The small output voltages of the bridges are directly amplified to processable values by signal conditioning circuits.

2.3.3 Overhead camera

An overhead camera\(^3\) is installed on the Watflex facility for tracking free floating objects and for gathering redundant position information of the robot. During this project, the camera was only used for validation purposes, \textit{e.g.}, to get absolute cartesian position data.

A frame grabber card\(^4\) is installed in the host PC (see Section 2.5) to acquire the camera data. A simple software\(^5\), delivered with the frame grabber card, is used to read the pictures from the camera and save them to hard disk. With the aid of an image processing software\(^6\), virtual reference points were superimposed with the grabbed picture to compare estimated positions with real ones. The accuracy of this measurement is limited by the resolution of the camera to $\pm 3\, \text{mm}$ (utilization of super-resolution would drop this number). The picture with the virtual reference points was grabbed prior to the experiments with test marks located at selected xy-positions and with a height offset of $188\, \text{mm}$, according to the height of the end-effector. This procedure assured accurate measurements without an otherwise necessary coordinate transformation from the spherical coordinates of the grabbed pictures to the cartesian coordinates of the glass surface.

2.4 Data acquisition

A data acquisition board\(^7\) forms the interface between the robot and the control unit. The board itself has a 12-bit A/D-converter with a maximum throughput of $100\, \text{kHz}$. Two 16 bit

\(^3\)Pulnix TM9701, 768 $\times$ 484Pixel@30fps, digital
\(^4\)Matrox Pulsar
\(^5\)Matrox Intellicam 2.06
\(^6\)Corel Photopaint 10
\(^7\)Burr Brown/Intelligent Instruments PCI-20098-2C
digital counters and 16 buffered digital input/output channels are available as well. The board is also equipped with two additional modules: The first is an analog input/output interface\(^8\) to supply the input signals for the amplifiers. It offers 12-bit D/A conversion with a throughput of 250\(K\) outputs/second. The second is a fast digital input/output board\(^9\) with a quadrature decoder capability to acquire the encoder data.

### 2.4.1 Strain measurements

The Watflex facility uses isolated strain gage amplifiers\(^10\) to transform small strain gage bridge voltages to a processable voltage range. These amplifier circuits, known as 'signal conditioning blocks', filter the measurements and provide also the supply voltage for the bridges. For each bridge, one signal conditioning block is necessary. A termination board carries these blocks and is the interface to the data acquisition card. All cables are shielded to suppress noise on the signals.

### 2.4.2 Angle measurements

Since the second data acquisition board module already provides a hardware quadrature decoder, the angle measurement is read through this device and directly converted into radians.

### 2.5 Control system

The setup of the control system is schematically shown in Figure 2.4. It consists of two personal computers that are connected by a 100\(M\)Bit TCP/IP local area network:

The host PC\(^{11}\) is the user interface. The control algorithms are programmed in Matlab/Simulink, compiled with the Matlab/Real-Time Workshop and then downloaded to

---

\(^8\)PCI-20003M-2

\(^9\)PCI-20007M

\(^10\)Intelligent Instrumentation PCI-5B38-05

\(^11\)Intel Pentium 4, 2.4\(GHz\), 1\(GB\) \(RAM\), Windows 2000 desktop PC, framegrabber card
the target PC. The overall control of the experiment and the analysis of the results is done on this PC. A second Simulink model runs on the host PC to provide a graphical user interface (GUI) for the controller during an experiment. With this GUI the user is able to tune parameters and read model values at arbitrary times. The host PC is also used for the image processing of the overhead camera video data.

The target PC\textsuperscript{12} runs the control algorithm uploaded from the host PC in real time. For most experiments a sampling time of 1\textit{ms} is used. The target PC is capable of showing frequently updated signals and model data on its own screen in graphical form or it sends this data to the host PC. The measured data is stored at each time step in the internal memory and can be uploaded to the host PC after finishing the experiment.

\textsuperscript{12}Intel Pentium, 200\textit{MHz}, 128\textit{MB RAM}, XPC-Target real-time operating system, data acquisition card
Chapter 3

Preparations

This Chapter shows the necessary preparations done to achieve the goals of this research project.

3.1 Determination of amplifier gains

Control algorithms for robots usually prefer to command a torque to each servomotor. The torque $\tau$ of a motor is approximately proportional to its input current $I$:

$$\tau = k_T I$$  \hspace{1cm} (3.1)

The torque constant $k_T$ is usually provided by the manufacturer of the motor. In order to command a torque, the knowledge of the conversion factor $k_A$ from the input voltage $V$ to the output current $I$ of the amplifier is necessary:

$$V = \frac{1}{k_A} \frac{1}{k_T} \tau$$  \hspace{1cm} (3.2)

The gain and the offset of each amplifier is adjustable with a potentiometer. Since one amplifier had to be replaced and different persons worked in the past on the Watflex facility, the offsets were adjusted and the amplifier gains were determined again. For the
gain determination, a DC-amperemeter\textsuperscript{1} was used to measure the steady state current, while a constant control voltage was applied to the input of the amplifier. The resulting maps are shown in Figure 3.1 and 3.2, where the circles denote the measurements and the straight line shows the linear regression of all measurements. The slopes of the straight line accord to the gain factors of the amplifiers. These were determined as $K_{SAMP} = 0.4948 \frac{A}{V}$ for the shoulder amplifier and $K_{EAMP} = 0.4679 \frac{A}{V}$ for the elbow amplifier.

![Figure 3.1: Voltage–Current map of amplifier for shoulder motor.](image)

### 3.2 Noise reduction in strain measurements

The +5V power supply for the termination board with the signal condition blocks for the strain gages, described in Section 2.4.1, is by default provided by the target PC (see Section 2.5). After encountering noise problems with more than two signal conditioning blocks installed, the power supply was switched to an external one. This reduced the noise

\textsuperscript{1}Tektronix DMM914 TrueRms
to a smaller value than the quantization noise of the data acquisition board output.

The actuator currents seem to introduce some other noise. A separation of the signal wires away from the power wires did not reduce this noise. The sources are probably the unshielded strain gages and the short wires connecting them to the bridge configuration. Since the noise is very low, no additional attempts to suppress it were made.

### 3.3 Calibration of strain gages

Figure 3.3 shows the labels and the exact location of the strain gage\(^2\) bridges mounted on the links. A calibration factor \(K_{ij}\) between voltage \(V_{ij}\) and strain \(\epsilon_{ij}\) at the bridge location \(ij\) had to be determined for each bridge.

The relation between strain \(\epsilon\) and static deflection \(w\) of the endpoint of a link is pro-

\(^2\)Micro Measurements CEA-13-250UW-350
\[ \epsilon(w) = \frac{3}{2} \frac{t}{l^3} [x_s - l]w \]  

where \( t \) is the thickness, \( l \) is the length of the link, and \( x_s \) is the strain gage location, measured from the root. This relation can be used to determine the calibration factors of the strain gage bridges.

The calibration setup is sketched in Figure 3.4: The beam was bent to a certain deflection \( w \) and the bridge voltages \( V_{ij} \) were measured. Because of its precision table, a milling machine was utilized for the bending experiments. The first end (18\( mm \)) of the beam was clamped, and the force was applied 18\( mm \) from the end; this accords to the clamping situation in the robot. The force was applied in the direction of gravity, which caused a small offset. Due to the setup, only one bending direction could be performed at a time, so separate experiments for each direction were necessary. Since the measurements were done in the university student shop with several high-power drives around, noise could
have a negative impact. To reduce this influence, the measurements were averaged over 100 Samples.

The analysis of the maps between deflection and voltage started with a linear regression for each bending direction separately to determine and subtract the offsets caused by gravity and by the bridge itself. Then the measurements were concatenated and the samples with zero deflection were deleted due to their critical accuracy. A second linear regression determined the slopes of the maps. The results are shown in Figure 3.5 and 3.6, where the symbols represent the measurements for each bridge, and the straight lines show their regressions. The measurements show a good linear behavior of the bridges with small absolute errors as shown in Figure 3.7 and 3.8. The errors in the first measurement are slightly dependent on the deflection. The most likely reason for that behavior is a slightly misplaced location of the applied force in one experiment. Since the relative error is below 1% the measurements are still considered valid. Appendix A.1 shows the results of similar
Figure 3.6: Static map of strain gage calibration for link two, clamped at 2A (○: bridge A; +: bridge B; -: according regression).

Figure 3.7: Absolute variance from linear regression of link one (—: bridge A; - - : bridge B; ··· : bridge C).
measurements done with the link clamped at the end and the force applied at the root. The error is in that case not depended on deflection. The slopes $s_{ij}$ of the static maps and Equation 3.3 are used to calculate a conversion factor $K_{ij}$ from voltage to strain (for link $i$, bridge $j$), see Equation 3.4 and 3.5. This calculation was done for each clamping situation separately to verify the conversion factors $K_{ij}$, see Table 3.1. Significant deviations between the two measurements occur in the cases where the location of the force applied was close to the pertaining strain gage bridges. Small misplacement of the force is probably the reason for the deviations. In order to proceed with the most accurate conversion factors, these were ranked according to their distance from the location of the applied force during each experiment; conversion factors with the same ranking were averaged. The final factors used, are shown in Table 3.2.
Link clamped near bridge A:

<table>
<thead>
<tr>
<th>Bridge $ij$</th>
<th>$s_{ij}$</th>
<th>$K_{ij}$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>$-216.299 \frac{V}{m}$</td>
<td>$2.034e - 4 \frac{1}{V}$</td>
<td>1</td>
</tr>
<tr>
<td>1B</td>
<td>$-115.248 \frac{V}{m}$</td>
<td>$1.983e - 4 \frac{1}{V}$</td>
<td>2</td>
</tr>
<tr>
<td>1C</td>
<td>$-10.068 \frac{V}{m}$</td>
<td>$1.690e - 4 \frac{1}{V}$</td>
<td>3</td>
</tr>
<tr>
<td>2A</td>
<td>$-155.597 \frac{V}{m}$</td>
<td>$2.017e - 4 \frac{1}{V}$</td>
<td>1</td>
</tr>
<tr>
<td>2B</td>
<td>$-82.399 \frac{V}{m}$</td>
<td>$1.978e - 4 \frac{1}{V}$</td>
<td>2</td>
</tr>
</tbody>
</table>

Link clamped near bridge C (or end):

<table>
<thead>
<tr>
<th>Bridge $ij$</th>
<th>$-s_{ij}$</th>
<th>$K_{ij}$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>$-9.600 \frac{V}{m}$</td>
<td>$1.772e - 4 \frac{1}{V}$</td>
<td>3</td>
</tr>
<tr>
<td>1B</td>
<td>$-113.638 \frac{V}{m}$</td>
<td>$2.011e - 4 \frac{1}{V}$</td>
<td>2</td>
</tr>
<tr>
<td>1C</td>
<td>$-214.129 \frac{V}{m}$</td>
<td>$2.055e - 4 \frac{1}{V}$</td>
<td>1</td>
</tr>
<tr>
<td>2A</td>
<td>$-6.659 \frac{V}{m}$</td>
<td>$1.823e - 4 \frac{1}{V}$</td>
<td>3</td>
</tr>
<tr>
<td>2B</td>
<td>$-82.767 \frac{V}{m}$</td>
<td>$1.969e - 4 \frac{1}{V}$</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3.1: Calculated conversion factors from voltage reading to strain.

Table 3.2: Final conversion factors.

\[
\begin{align*}
    w_{ij} & = \frac{1}{s_{ij}} V_{ij} \\
    K_{ij} & = \frac{\epsilon_{ij}}{V_{ij}} = \frac{3}{2} \left[ x_{s_{ij}} - l_{ij} \right] \frac{1}{s_{ij}}
\end{align*}
\]
Chapter 4

Friction in joints

Friction is inevitable in most technical systems. It occurs between surfaces which are in contact and have relative motion, e.g., bearings, transmissions, hydraulic and pneumatic cylinders, valves, brakes and wheels. Lubricants, like oil or grease, are often used to reduce friction effects. Although sometimes a desirable property, like for brakes, friction usually reduces the performance in most technical applications, can lead to limit cycles and result in steady state errors. Friction is also highly non-linear. The friction phenomenon has been studied for many years and is still a research objective, see [7, 8].

This chapter investigates several phenomena of friction and shows their relevance in robotic actuators with harmonic drive gearing. It introduces several friction models and presents their utilization for friction compensation. The chapter contains experiments and results showing the practical application of friction compensation on harmonic drive actuators.

4.1 Friction phenomena

The tangential reaction force between two surfaces in relative motion is called friction. As a result of many different mechanisms, these reaction forces depend on the properties of the bodies and their environment. Especially surface conditions, materials, displacements,
relative velocities and lubricants all influence the friction behavior.

### 4.1.1 Flat contact between surfaces

The contact between flat, dry surfaces can be modelled as elastic and plastic deformation forces of microscopic asperities in contact. Even without a lubricant, a thin oxide film will grow on the surface of commonly used materials like steel, forming a boundary layer, see Figure 4.1a. Since this layer has a lower shear strength than the body material, most shearing will occur in the boundary layer. In lubricated contact, the boundary layers are formed by a reaction between the body surface and additives to the bulk oil. Boundary layer thickness varies from a few atomic thicknesses to a fraction of a micron [8].

![Figure 4.1: Contact between engineering materials:](image)
a.) dry, b.) partially lubricated, c.) fully lubricated.
4.1.2 Rolling contact between surfaces

In dry rolling contact, friction is caused by a non-symmetric pressure distribution between the bodies. The pressure distribution is a result of elastic hysteresis in either of the bodies, or local sliding in the contact.

4.1.3 Lubricated contact – velocity dependence

In presence of lubrication, additional physical mechanisms influence the friction. The friction force is mainly a function of the sliding velocity. There are four different dynamic regimes, schematically shown in Figure 4.2:

Regime I, static friction, is not dependent on velocity. The contact of two bodies occurs at the asperity junctions shown in Figure 4.1a. An applied shear force causes an elastic deformation of the junctions, which leads to pre-sliding displacement. A plastic deformation of both, the boundary layer and the asperities causes a rise of static friction (often called stiction).

Regime II, boundary lubrication, covers the dynamic behavior at very low velocities. The lubricant forms a surface film – a boundary layer as described in section 4.1.1. The friction force is determined by the shear strength in this layer.

Regime III, partial fluid lubrication, leads to a fluid layer of lubricant between the two bodies at higher velocities due to hydrodynamic effects, see Figure 4.1b. The friction force is determined by the shear force in the fluid layer. It will be lower than the friction force in the low velocity case; this is called the Stribeck effect.

Regime IV, full fluid lubrication, characterizes a complete separation of the bodies due to high velocities, see Figure 4.1c. Hydrodynamic effects become significant and the friction force will increase with velocity.
CHAPTER 4. FRICTION IN JOINTS

4.1.4 Frictional lag

During unidirectional movements, friction shows also dynamic behavior: A delay exists between a change in velocity and the corresponding change in friction, as shown in Figure 4.3 – this is called frictional lag.

Figure 4.3: Frictional lag: The friction–velocity relation.
4.1.5 Pre-sliding displacement

Another dynamic behavior is the pre-sliding displacement, mentioned in Canudas de Wit et al. [6]. A force applied to two surfaces in contact, which is smaller than the break-away force, will cause a relative displacement. This displacement is elastic with hysteresis, as Figure 4.4 displays.

![Pre-sliding displacement](image)

Figure 4.4: Pre-sliding displacement.

4.1.6 Dependency of static friction on dwell time

Some publications (e.g., [8]) mention a behavior referred to as dwell time on the break-away force. The break-away force increases with the dwell time, as shown in Figure 4.5.

4.2 Friction in Harmonic drive gearing

The harmonic drive gearing in the actuators causes a major part of its total friction due to its construction. Solid friction in the flexspline and friction due to the large area of toothing, compared to those of other types of gearing, are the main reasons. The roll
bearing between the wave generator and the flexspline causes another part of the friction due to its comparably high radius. In addition to that, friction caused by the DC-motor brushes and all other bearings is also present. The overall friction behavior is strongly nonlinear and and similar to many friction phenomena described above. Friction is also dependent on many factors like the current state of the motor (e.g., velocity, absolute position) and its environment (e.g., temperature). The observations of the Watflex joints are described in 4.5.1.

4.3 Friction models

A friction model is usually necessary to take friction effects into account or compensate friction in servo control applications. In the past, a wide range of models for friction forces have been developed, see [1, 3, 7, 8, 11, 12]. They differ in the number of covered friction phenomena and complexity.

4.3.1 Static models

Static friction models are usually maps between velocity and friction force. They are called "static" because there are no state variables nor differential equations present in
these models. Their complexity rises with the number of covered friction phenomena. The following sections describe some of the most common static friction models.

**Classical models**

The classical models consist of different components, each of which takes care of certain aspects of the friction force. The basic idea is that friction opposes motion, and that the friction force is independent of velocity and contact area. It can be described as

\[
F = F_C \text{sgn}(v),
\]

where \( F_C \) is proportional to the normal load \( F_N \), i.e. \( F_C = \mu F_N \). This description is often referred as **Coulomb friction**. The friction at zero velocity is not specified and may take on any value in the interval \([-F_C, F_C]\), depending on how the sign function is defined. Because of its simplicity, the Coulomb friction model has been often used for friction compensation, see Figure 4.6a.

This friction model can be extended with a friction force dependent on velocity, as shown in Figure 4.6b. This extension, called **viscous friction**, is described by

\[
F = F_v v + F_C \text{sgn}(v)
\]

in the simple case, or by

\[
F = (F_v |v|^\delta v + F_C) \text{sgn}(v)
\]

with a nonlinear dependency on \( v \). \( \delta_v \) is a parameter used to fit the extended model better to experimental data than the simple model.

**Stiction**, static friction, describes the friction force at rest. It is modelled as a function of the external force \( F_e \) and the static limiting force \( F_S \).

\[
F = \begin{cases} 
F_e & \text{if } v = 0 \text{ and } |F_e| < F_S \\
F_S \text{sgn}(F_e) & \text{if } v = 0 \text{ and } |F_e| \geq F_S 
\end{cases}
\]
Since stiction is not only dependent on the velocity $v$ but also dependent on $F_e$, it cannot completely modelled as static map between velocity and friction force. If such a map is desired, stiction must be expressed as a multi-valued function that can take on any value between the two extremes $-F_S$ and $F_S$. Specifying stiction in this way leads to non-uniqueness of the solutions to the equations of motion for the system, see Bliman and Sorine [1].

Different combinations of the friction models described above are referred to as a classical model, see Figure 4.6c. These models have either linear in velocity or constant components. Stribeck [17] observed that the velocity dependence is continuous as shown in Figure 4.6d. This is referred to as Stribeck friction.

A more general description of friction is

$$F = \begin{cases} 
F(v) & \text{if } v \neq 0 \\
F_e & \text{if } v = 0 \text{ and } |F_e| < F_S \\
F_S \text{sgn}(F_e) & \text{otherwise}
\end{cases}$$

(4.5)
where \( F(v) \) is an arbitrary function, which may look as in Figure 4.6d. This function is often asymmetrical.

**The exponential model**

A common choice for the function \( F(v) \) is

\[
F(v) = \left( \alpha_0 + \alpha_1 e^{-\left(\frac{v}{v_S}\right)^\delta} \right) \text{sgn}(v) + \alpha_2 v. \tag{4.6}
\]

This model will be later referred to as the *exponential friction model*. It covers Coulomb, viscous, static and Strubeck friction and is characterized by only a few parameters:

- \( \alpha_0 \)  Coulomb friction \( F_C \)
- \( \alpha_1 \)  Additional stiction force \( F_S - F_C \)
- \( \alpha_2 \)  Viscous friction coefficient \( F_v \)
- \( v_S \)  Strubeck velocity
- \( \delta \)  Form factor

Usually, different sets of parameters are used for either direction. Figure 4.7 shows the static map of the exponential model with different parameter sets for either direction.

**The Karnopp model**

A main disadvantage of a model such as Equation 4.5 for simulations or control purposes, is the problem of detecting when the velocity is zero. A friction model presented by Karnopp [11] was developed to overcome these problems and to avoid switching between different equations for sticking and sliding. The model defines a zero velocity interval \( |v| < DV \) which works as a dead-zone and maintains a virtual zero velocity when the real velocity is within this interval. Depending on whether \( |v| < DV \) is true or not, the friction force is either a saturated version of the external force or an arbitrary static function of
The drawback is that it is dependent on the characteristics of the rest of the system. The external force is an input to the model, but it is not always explicitly given. The model therefore has to be adapted to each particular system.

### 4.3.2 Dynamic models

Dynamic friction models have one or more internal states and can cover more friction phenomena than the static models. But the complexity and the implementation effort is usually higher for dynamic models than it is for static ones.

**Dahl model**

The *Dahl model* [3] was based on Dahl’s findings that friction in ball bearings behaves similarly to solid friction. The model was developed for the purpose of simulating control systems with friction, especially systems with bearing friction. It covers, in addition to Coulomb friction, dynamic effects like pre-sliding displacement.

Friction $F$ is modelled as function of displacement $x$ by the following differential equa-
CHAPTER 4. FRICTION IN JOINTS

\[ \frac{dF}{dx} = \sigma \left(1 - \frac{F}{F_C} \operatorname{sgn}(v)\right)^\alpha \]  

(4.7)

\(F_C\) is the Coulomb friction force, \(\sigma\) is the stiffness coefficient, and \(\alpha\) is a form factor (a common choice is \(\alpha = 1\)). \(\sigma\) is also determined by the slope of the stress-strain curve at rest, as shown in Figure 4.8. The form factor \(\alpha\) shapes the stress-strain curve – higher values will give the curve a sharper bend. An important property of this model is its rate independence, since the friction force is only a function of the displacement and the sign of velocity. Assumed that the initial condition satisfies \(|F(0)| < F_C\), \(F\) is never larger than \(F_C\), which means that the Dahl model does not cover stiction effects. For the common case \(\alpha = 1\), the model is

\[ \frac{dF}{dt} = \sigma \left(v - \frac{F|v|}{F_C}\right). \]  

(4.8)

To make it easy to compare the Dahl model to the models below, the friction force can be chosen as \(F = \sigma z\):

\[ \frac{dz}{dt} = v - \frac{\sigma|v|}{F_C} z \]

\[ F = \sigma z, \]

(4.9)

**Bliman/Sorine model**

Bliman and Sorine [1] proposed a class of dynamic friction models, which are also rate independent. The friction force depends only on the sign of the velocity and on the space
variable \( s \), that is defined as
\[
s = \int_0^t |v(\tau)| d\tau.
\] (4.10)

In this space variable \( s \), the friction model can be expressed as
\[
\frac{dx_s}{ds} = Ax_s + Bv_s
\]
\[
F = Cx_s
\] (4.11)

The variables \( A, B, C \) can be scalars for a first order friction model or they have entries similar to a second order state-space model.

The first order model is given by
\[
A = \frac{-1}{\epsilon_f}, \quad B = \frac{f_1}{\epsilon_f}, \quad \text{and } C = 1,
\] (4.12)

which can be also written as
\[
\frac{dF}{dt} = \frac{dF}{ds} \frac{ds}{dt} = |v| \frac{dF}{ds} = \frac{f_1}{\epsilon_f} \left( v - |v| \frac{F}{f_1} \right).
\] (4.13)

With \( f_1 = F_C \) and \( \epsilon_f = \frac{f_1}{\sigma} \), this model is identical to the Dahl model with \( \alpha = 1 \), see Equation 4.8. Hence it does not cover stiction behavior.

The second order model can achieve a stiction peak for starting motions:
\[
A = \begin{pmatrix}
\frac{-1}{\eta \epsilon_f} & 0 \\
0 & \frac{-1}{\epsilon_f}
\end{pmatrix}
\]
\[
B = \begin{pmatrix}
f_1 \\
\frac{f_1}{\eta \epsilon_f} \\
\frac{-f_2}{\epsilon_f}
\end{pmatrix}
\] and \( C = (1 \ 1) \) (4.14)

Schematically, these are two parallel Dahl models, a fast and a slow one. The fast model has higher steady-state friction than the slow model; the subtraction from each other results in a stiction peak. It should be noted, that this peak only occurs during the start of a motion, but real friction would have this peak also at decelerating motion as observed by
Four parameters have to be determined: $f_1$, $f_2$, $\epsilon_f$, and $\eta$. According to Bliman and Sorine [1], these can be obtained by determining related parameters of the transient curve, sketched in Figure 4.9.

![Hysteresis curve of the Bliman/Sorine model.](image)

**Figure 4.9: Hysteresis curve of the Bliman/Sorine model.**

**LuGre model**

The Lund-Grenoble (LuGre) model proposed by Canudas de Wit et al. [6] is a dynamic friction model that combines the stiction behavior and the Dahl effect, with arbitrary steady state friction characteristics like Coulomb, viscous friction and the Stribeck effect. It is developed from the idea of modelling surface asperities (see section 4.1.1) as bristles. A tangential force applied between two surfaces in contact will deflect the bristles like springs and increase the friction force. A sufficiently large force will cause the deflected bristles to slip. The idea covers the pre-sliding displacement and the elastic effect during changes of the direction of motion. To model the irregular form of surfaces, the bristles are distributed randomly. The LuGre model introduces the state variable $z$ which is modelled
as the average bristle deflection

\[
\frac{dz}{dt} = v - \frac{\sigma_0}{g(v)} |v| z, \tag{4.15}
\]

where \(v\) is the relative velocity between the two surfaces, and \(\sigma_0\) is the stiffness. The function \(g\) is positive and depends on many factors like lubrication, material properties, and temperature. To cover the Stribeck effect it will decrease monotonically from \(g(0)\) when \(v\) increases. The friction force is described as

\[
F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \alpha_2 v \tag{4.16}
\]

where the first two terms denote the force generated by the bristle deflection, and the last term denotes the viscous friction. \(\sigma_1\) is a damping coefficient and \(\alpha_2\) the viscous friction coefficient.

The LuGre model given by Equation (4.15) and (4.16) is characterized by the function \(g(v)\) and the parameters \(\sigma_0, \sigma_1\) and \(\alpha_2\). To account for the Stribeck effect, [5] proposes

\[
g(v) = \alpha_0 + \alpha_1 e^{-\left(\frac{v}{v_0}\right)^2} \tag{4.17}
\]

as parametrization of \(g(v)\). It will be shown in section 4.5.4 that this choice of \(g(v)\) implies a static map similar to the exponential friction model described in Section 4.3.1.

The investigation of the dynamical behavior in [6] shows that several characteristics of friction are modelled: pre-sliding displacement, frictional lag, varying break-away force, and stick-slip motion.

### 4.4 Friction compensation

This section presents briefly the concepts for utilization of friction models to compensate friction. It mentions issues of a friction compensation and shows different ways of possible
implementations. The presented friction compensations are restricted to those which are applicable to the Watflex joints, i.e. no output torque measurements or encoders mounted on the output shaft are used. Since the motor torque is proportional to its input current, the torque constant is already included in the “Robot Joint”-block, therefore the input for this block is the motor torque in the following.

Figure 4.10 describes a visualization of a real robot joint. It can be seen as an ideal motor with the subtracted friction.

![Figure 4.10](image)

Figure 4.10: Approximating the real robot joint by an ideal motor minus friction.

### 4.4.1 Feed-forward map

The simplest way to compensate friction in servo drives is a feed-forward map as Figure 4.11 displays. A friction torque $\tau_f(\tau)$ is added to the input torque $\tau$ to add an offset to the input signal for the motor depending on the sign of the input. In the ideal case, this offset should be exactly the friction torque. This will never happen in practice, because friction is, as mentioned in Section 4.2, strongly dependent on the current state of the motor and its environment. Therefore, the added offset should always be smaller than the real friction of the motor (undercompensation) to avoid instabilities due to the friction compensation.

The capability of this feed-forward compensation is limited to the reduction of the Coulomb friction. It cannot compensate stiction effects, viscous friction, and does not
provide back drivability to the motor, since there is no feed-back information from the motor. Because of these properties, the application is limited to direct feedback control loops, like PID-control. More sophisticated control algorithms provide motor torques as a control signal which should be similar to the real output torque of the motor and if they do not fulfill the internal model principal (see Lunze [14]), this friction compensation would not prevent large steady state errors. This friction compensation also increases the non-linearities of the motor, because it will switch the sign of the offset based on the input signal only to changes in the direction of rotation.

Usually, the compensation map is just the Coulomb friction model as shown in Figure 4.12a. The friction torque $t_f$ has always the same sign as the input signal and can be unsymmetric. The slope for a zero input is infinite, which causes usually a chattering of the motor input torque when the friction compensation is used in a direct feedback loop. Since this chattering will decrease the lifetime of the amplifier and the motor, it should be avoided or at least strongly reduced.

A remedy would be a decreased slope at zero input. This was done by Shi et al. [15] for a PID position control. The map is shown in Figure 4.12b. But the steady-state error of the system can increase with this friction compensation, due to the strong under compensation of the friction at low velocities.
Figure 4.12: Static feed-forward maps:
a.) Coulomb friction; b.) Coulomb friction with finite slope around zero.
4.4.2 Combinations of feed-forward and feed-back

An extension of the feed-forward friction compensation is shown in Figure 4.13. Part 1 is a feed-forward compensation as described previously, but Part 2 is an additional velocity feed-back map. This map provides a compensation for viscous friction and can include the Strubeck effect. A compensation of the stiction force is theoretically possible, but in practice not applicable, because an infinite slope of both compensation parts for zero velocity would cause chattering. Reducing this slope would result in a zero velocity reading and therefore prevent any feed-back compensation.

This type of friction compensation introduces also an increased non-linearity as in the pure feed-forward case.

4.4.3 Velocity feed-back configuration

Most of the more sophisticated friction compensations use a velocity feedback configuration, as shown in Figure 4.14.

This configuration is similarly used for static and dynamic friction models because the velocity is usually the input signal for a friction model. This friction model estimates a friction torque and adds it to the input signal for the motor. It is possible to cover all the mentioned static friction effects and also dynamic phenomena. Back drivability, which is often desired for the motors can also be assured.
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This configuration has one major drawback: as already mentioned before, the starting of the motor from rest needs an input torque which is higher than the stiction torque of the motor. This torque should be provided by the friction model, but the friction model will not be in effect until the motor is moving. This behavior is especially of interest for actuators with harmonic drive gearing, because the ratio between stiction torque and rated output torque is quite high (about $\frac{1}{5}$ for the Watflex joints). For control purposes, the actual working torques are often far below the rated output torques and get close to zero with reaching the control objective. For these reasons, a modification to the friction compensation scheme has to be done.

4.4.4 Qualified velocity feedback

A modification of the velocity feed-back scheme is displayed in Figure 4.15. It was devel-

Figure 4.14: Velocity feed-back friction compensation scheme.

Figure 4.15: Qualified velocity feed-back scheme.
oped during the project and eliminates the drawbacks of a pure velocity feed-back similar to that mentioned in the previous section. The velocity feed-back is preserved, but the velocity signal is modified around zero, based on the current input signal. This configuration is later referred as *qualified friction compensation*. The term “qualified” was used to emphasize the qualification of the velocity signal.

The idea to include, in addition to the current speed, the input signal to estimate the friction torque was realized by a cross-fading between the velocity and the input signal for velocities around zero, see Figure 4.16. The modified signal $\tilde{\theta}$ is then used instead of the pure velocity for the friction compensation. Depending on the current speed, the ratio between torque input and current velocity is determined. The saturation of the input signal $\tau \in [-\delta, \delta]$ guarantees that the modified velocity $\tilde{\theta}$ causes never a higher magnitude of the friction model output $\tau_f$ than the real friction torque when the signals are mixed.

![Figure 4.16: Qualified friction model scheme.](image)
The mixing ratio $\gamma$ is generated by an amplified and saturated velocity $\dot{\theta}$:

$$\gamma = \begin{cases} 
  k_\gamma |\dot{\theta}| & \text{for } k_\gamma |\dot{\theta}| < 1 \\
  1 & \text{for } k_\gamma |\dot{\theta}| \geq 1
\end{cases} \quad (4.18)$$

The speed value where the friction model is fully based on the real velocity is set with the factor $k_\gamma$. The gain $k_\tau$ takes care of the conversion from input torque to pseudo-speed $\dot{\theta}_\tau$. The mixing of the two signals is linear, i.e.

$$\tilde{\dot{\theta}} = \gamma \dot{\theta} + (1 - \gamma) k_\tau \tau \quad (4.19)$$

This qualified friction compensation is now not only capable of covering all friction phenomena that the pure velocity configuration does, but it is also able to compensate the friction for input signals with smaller magnitude than the stiction force.

The only drawback of this configuration is that the parameters $k_\gamma, k_\tau$ and the saturation limit $\pm \delta$ have to be adapted for each system, but this can be done easily.

4.5 Experiments and results

The experiments described in this section are performed on both actuators of the Watflex robot. Since the results between both are similar, only the experiments with the shoulder actuator are shown in this section. The experiments with the elbow actuator can be found in Appendix A.2

4.5.1 Properties of the actuator

Initial qualitative experiments with the shoulder actuator showed that it has, as expected, a lot of internal friction. It was noticed, that static friction components like Coulomb, static, viscous and Stibbeck friction are present, together with dynamic friction components, especially pre-sliding displacement. There was also an influence of the absolute motor angle
and the temperature on the friction behavior. Additionally, a sticking of the motor occurred sometimes after a long rest for slowly increasing input signals – the break-out torque was then in some cases higher than twice the stiction torque.

The technical specifications of the shoulder motor can be found in Table 4.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>RFS-32-6030</td>
</tr>
<tr>
<td>Rated Output Torque</td>
<td>50Nm</td>
</tr>
<tr>
<td>Rated Output Speed</td>
<td>60rpm</td>
</tr>
<tr>
<td>Maximum Output Torque</td>
<td>220Nm</td>
</tr>
<tr>
<td>Torque Constant</td>
<td>13.3 $\frac{Nm}{A}$</td>
</tr>
<tr>
<td>Inertia at Output Shaft</td>
<td>3.1 kgm$^2$</td>
</tr>
<tr>
<td>Gear Ratio</td>
<td>1 : 50</td>
</tr>
<tr>
<td>Mass</td>
<td>11.8kg</td>
</tr>
</tbody>
</table>

Table 4.1: Technical specifications of the shoulder motor.

The evidence for the static friction behavior of the joint, gathered by experiments, is provided in Section 4.5.4. For the proof of the existence of dynamic friction effects, the pre-sliding displacement of the actuator was investigated. A triangle shaped, periodic input signal with a magnitude of about 75% of the Coulomb friction force and a slope of 2.4 $\frac{Nm}{s}$ was applied. The resulting pre-sliding displacement is shown in Figure 4.17. In this plot, four measurement periods are shown. The multiple lines at some parts of the curve are caused by measurement and quantization noise. The graph shows, that an elastic pre-sliding displacement with hysteresis is present, similar to the observations made by Canudas de Wit et al. [6] and sketched in Figure 4.4.

### 4.5.2 Requirements for friction compensation

The frictional behavior of the actuator shows that friction compensation will be necessary for good performance of most control algorithms. For good compatibility with different
control strategies, several requirements had to be fulfilled. Especially the compensation of the non-linearity was of interest, \textit{i.e.} to get a linear transfer function for the actuator, because model based controllers could be designed with linear control theory. The requirements were in detail:

- **Linear behavior** – The friction compensated motor should have a linear transfer function, either \( G(s) = \frac{1}{Js^2 + Bs} \) with viscous friction in the model or \( G(s) = \frac{1}{Js^2} \) as ideal motor model.

- **Good behavior for small input signals** – The desired output torque of the motor might be smaller than the break-out torque.

- **Prevent chattering** – Chattering of the control signal for the motor might reduce the lifetime of the actuator and the amplifier.

- **Assure back drivability** – This is necessary for the linear behavior.
• **Independent from the control algorithm** – It should be possible to apply the friction compensation to different control algorithms without changes to it.

• **Saving resources** – The speed of the control algorithm should not be slowed down significantly, and the friction compensation should contain memory usage.

To fulfill all of the mentioned requirements, several approaches with different friction models and compensation schemes were made; the major steps are described in the sections below.

### 4.5.3 Choosing a model

Since the motors showed dynamic as well as static friction behavior the first approach to compensate friction was the utilization of a dynamic friction model. Two dynamic models were considered: The Bliman/Sorine model and the LuGre model.

During initial experiments, it turned out that the transient curve (shown in Figure 4.9) of the Bliman/Sorine model was quite difficult to measure exactly. The limitation in resolution of the encoders was a major reason for that. Other issues were encountered by Gräfert [9] who describes a comparison between the Bliman/Sorine model and the LuGre model. The advantages of the LuGre model were a better performance at zero-velocities, no dependency on the trajectory, and a more reliable parameter determination.

Because of the initial experiments and its widespread application, the LuGre model was chosen for further investigation.
4.5.4 Friction compensation with the LuGre model

Utilizing the LuGre model, a DC-motor with friction is given by

\[
\begin{align*}
J \frac{d^2 \theta}{dt^2} &= \tau - \tau_f \\
\frac{dz}{dt} &= \dot{\theta} - \frac{\sigma_0}{g(\dot{\theta})} |\dot{\theta}| z \\
g(\dot{\theta}) &= \alpha_0 + \alpha_1 e^{-\left(\frac{\dot{\theta}}{v_S}\right)^2} \\
\tau_f &= \sigma_0 z + \sigma_1 \frac{dz}{dt} + \alpha_2 \dot{\theta},
\end{align*}
\]

(4.20)

where \( J [kgm^2] \) is the moment of inertia of the motor, \( \theta [rad] \) is the angular position of the motor, \( \tau [Nm] \) is the torque of the ideal motor, \( \tau_f [Nm] \) is the friction torque, and \( z [rad] \) is the microscopic deformation of the bristles. For friction compensation, the parameters of the model have to be found by matching a simulated motor output (generated with the scheme shown in Figure 4.10) with the real output when applying the same input signal to both. The parameter estimation for the LuGre model was done in two steps, see Canudas de Wit and Lischinsky [5]: The static parameters of the model were determined first, and by using these, the remaining dynamic parameters were found. Table 4.2 shows the static and dynamic parameters with their units and their interpretation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>( Nm )</td>
<td>Coulomb friction</td>
</tr>
<tr>
<td>( \alpha_1 + \alpha_0 )</td>
<td>( Nm )</td>
<td>Static friction</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>( \frac{Nm}{rad^2} )</td>
<td>Viscous friction</td>
</tr>
<tr>
<td>( v_S )</td>
<td>( \frac{rad}{s} )</td>
<td>Striebeck velocity</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>( \frac{Nm}{rad} )</td>
<td>Stiffness coefficient of microscopic deformations</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>( \frac{Nm}{rad} )</td>
<td>Damping coefficient of microscopic deformations</td>
</tr>
</tbody>
</table>

Table 4.2: Parameters of the LuGre model.
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Static parameter estimation

The static parameters of the LuGre model $\alpha_0$, $\alpha_1$, $\alpha_2$, and $v_S$ can be estimated by construction of the friction–velocity map measured during constant velocity rotation. In the steady-state case ($\frac{d\theta^2}{dt^2} = 0$, $\frac{d\dot{z}}{dt} = 0$) the friction torque reduces to

$$
\tau_{ss} = (\alpha_0 + \alpha_1 e^{-\left(\frac{\dot{\theta}}{v_S}\right)^2}) \text{sgn}(\dot{\theta}) + \alpha_2 \dot{\theta},
$$

which is similar to the exponential friction model, mentioned in Section 4.3.1 (page 27).

Measurement of the friction–velocity map The friction torque was measured in a closed loop experiment under velocity PI control, as shown in Figure 4.18. Because of the temperature dependency of the harmonic drive gear, the experiment was designed to scan the entire speed range from $0.61 \text{ rad/s}$ to $0.0004 \text{ rad/s}$ in one run. During this run the rolling direction had to be changed several times, because the shoulder motor was restricted to 3 turns in either direction due to its supply cables. Each desired speed was held constant for a certain period. The first half of the period was used to give the system time to settle in the steady-state motion. During the second half of each period the means of all input torque and output speed values were calculated to eliminate noise. The step size was varied with the speed, because more data points were needed at very low velocities in order to get a good parameter estimation in this region. The time for each step was also speed dependent to reduce scatter of the measurements in the low velocity region, where the quantization noise had a significant impact. The shorter periods at faster velocities kept the motor within the absolute angle restrictions and avoided unnecessary data processing.
and running time. With this configuration, one run took 20 minutes and sampled the data every 1ms.

The velocity trajectory with all above mentioned features is shown in Figure 4.19a. In order to achieve the velocity tracking over the three speed decades, a PI controller with variable gains was designed. During preparatory experiments, good PI-parameters were found for certain desired speeds. To apply nearly optimal PI-parameters between these speeds, third and fourth order polynomials were used to generate suitable parameters for each desired speed. The parameter changes are shown in table 4.19b,c – their changes are only applied together with desired speed changes.

The resulting static friction–velocity map is shown as dots in Figure 4.20. As desired with the procedure described above, more data points are collected at slow velocities.
Parameter optimization  Different parameter sets for each direction were then determined for the steady-state friction torque $\tau_{ss}$, see Equation 4.21. This was done by using a non-linear optimization of the four parameters, using the optimization toolbox of Matlab. The command `fminsearch`, which performs a multi-variable simplex search, provided good results, as shown with a dashed line in Figure 4.20. The initial values for the search algorithm were determined by inspection of the measured map. Figure 4.21 and Figure 4.22 show an enlarged view on the low velocity regions. The resulting static parameters are collected in Table 4.3.

The found parametrization reflects the real measurements very well. The main reasons for small deviations between the measurements are sometimes not avoidable stick-slip motion in the very low velocity domain and cable drag in the higher velocity domain. Different sets of static parameters for each rotating direction were necessary due the the unsymmetrical behavior of the actuator.
Chapter 4. Friction in Joints

Figure 4.21: Static friction–velocity map, zoomed on positive stiction region (●: measurement; -: parametrization).

Figure 4.22: Static friction–velocity map, zoomed on negative stiction region (●: measurement; -: parametrization).
Dynamic parameter estimation

With the static parameters given, the two dynamic parameters $\sigma_0$ and $\sigma_1$ can be determined. Open-loop experiments including zero crossings of the velocity were performed to enhance the dynamic friction effects. The recorded data was then used to search for a $\hat{\sigma} = [\hat{\sigma}_0, \hat{\sigma}_1]$ that minimizes the error cost function

$$E\{\theta, \theta_m; \hat{\sigma}\} = \sum_{k=0}^{N} \left[ \theta(k, \sigma) - \theta_m(k, \hat{\sigma}) \right]^2$$  \hspace{1cm} (4.22)

where $\theta(k, \sigma)$ is the $k^{th}$ sampled actuator angle and $\theta_m(k, \hat{\sigma})$ is the $k^{th}$-value of the model output position, see Figure 4.23. The LuGre model

$$J \frac{d^2 \theta_m}{dt^2} = \tau - \dot{\tau}_f$$

$$\frac{dz}{dt} = \dot{\theta}_m - \frac{\hat{\sigma}_0}{g(\dot{\theta}_m)}|\dot{\theta}_m|z$$

$$g(\dot{\theta}_m) = \alpha_0 + \alpha_1 e^{-\left(\frac{\dot{\theta}_m}{v_S}\right)^2}$$

$$\dot{\tau}_f = \hat{\sigma}_0 z + \hat{\sigma}_1 \frac{dz}{dt} + \alpha_2 \dot{\theta}_m,$$

was implemented in Simulink with a sample time of 1ms. The moment of inertia $J = 3.41 \text{kgm}^2$ of the motor was determined by a procedure described below in section 4.5.6.

The attempt to use a simplex search for estimating the dynamic parameters, resulted either in instabilities and a lack of robustness, or the parameter $\sigma_1$ was running towards...
zero. According to Canudas de Wit et al. [6] $\sigma_0$ should be very high, but this leads to an unstable model in this discrete case. Finally, the parameters were searched manually, because $\sigma_1$ must be greater than zero [6]. It turned out, that the static friction components were prevailing, i.e. the dynamic parameters had not much influence on the overall friction model. However, with the choice of $\hat{\sigma} = [259 \, \text{Nm} \, \text{rad}^{-1}, 10 \, \text{Nms} \, \text{rad}^{-1}]$ the agreement between the simulated and the real actuator response was quite good, as it is shown in Figure 4.24.

Model validation

The validation of the LuGre friction model with the determined parameters was performed under friction compensation. If friction can be exactly predicted, the system under friction compensation would behave like an ideal motor with no energy dissipation. In a P-controlled closed loop experiment, an ideal system would behave as an oscillator without friction. Therefore, small imprecisions of the friction model would lead either to limit cycle extinction if friction is underestimated (energy is dissipated) or to an unstable behavior if friction is overestimated (energy is supplied).
Figure 4.24: Dynamic parameter estimation experiment: Comparison of real and simulated angle.
The experimental setup was implemented according to the qualified friction compensation, see section 4.4.4, with the following parameters: \( k_\gamma = \frac{1}{0.01} s^{-1} \), \( k_\tau = 1 \text{rad} \text{Nms}^{-1} \), \( \delta = \pm 0.5 \text{rad} \text{s}^{-1} \). The controller for the validation setup had a P-gain of \( 5 \frac{\text{Nm}}{\text{rad}} \), and the step input at 1s had a magnitude of 1rad. The step response of the friction compensated system is shown in Figure 4.25 as a solid line; additionally a dashed line shows the simulated step response. The damping of the measured signal indicates a small undercompensation of joint friction, but this property is desired for the final friction compensation. A more serious issue is the sticking of the motor at velocity reversals. This can be explained by a too strong modelling error in the dynamic domain. The discrete nature of the system limits the range of possible dynamic parameters.

Since this friction compensation does not fulfill the requirements, it is not considered feasible.
4.5.5 Friction compensation with the exponential model

The exponential friction compensation was developed based on the observations, that the static parameters of the LuGre model had by far more influence on the estimated friction than the dynamic ones. This friction compensation, described in Paragraph 4.3.1, covers only static friction effects. As mentioned before, the exponential model is equal to the static part of the LuGre model, i.e. the static parameters of the LuGre model can be used for the exponential model. To get a smooth velocity measurement $\dot{\theta}$, the angle signal was filtered by a second order digital Butterworth filter with a cutoff frequency of 10 Hz.

Implementation of the exponential model

A little modification had to be made to the friction–velocity map: Since the map had an infinite slope at zero velocity, it was very sensitive to the smallest deformations of the motor and measurement noise. Therefore, the slope was decreased by multiplying each side of the map by

$$1 - e^{-|\dot{\theta}|k_s},$$

(4.24)

where $k_s$ is a factor used to adjust the slope. Decreasing the slope reduces the modelled stiction and therefore the performance of the friction compensation. Considerations between friction compensation performance and suppression of chattering of the friction torque signal were made to determine the factor $k_s$. The factor $k_s = 300$ was chosen by qualitative experiments and used for all further experiments. The resulting map is shown in Figure 4.26.

Model validation

This model was validated in the same way as the LuGre model. The gain of the P-control loop was again $5 \frac{Nm}{rad}$, and the step was performed at 1 s with a magnitude of 1 rad. Only the parameters for the qualified friction compensation setup were adapted to this friction model: $k_\gamma = \frac{1}{0.01} \frac{s}{rad}$, $k_\tau = \frac{rad}{Nms}$, $\delta = \pm 0.01 \frac{rad}{s}$. The step response is shown in Figure 4.27.
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Figure 4.26: Exponential friction compensation map.

Very good agreement between the friction compensated actuator and an ideal motor model is given with this friction compensation. The required under compensation is given with this model, like for the LuGre model. The plot of the estimated friction torque in Figure 4.28 indicates no chattering or ripples which was one of the requirements of the friction compensation. Since this model is implemented in the qualified friction compensation scheme, it is also able to capture the friction effects for small inputs. To prove the back drivability experimentally, the strain gage readings, which are proportional to the applied torque, are plotted together with the resulting actuator speed in Figure 4.29. As expected for an ideal system, a torque pulse leads to a change in velocity; if no torque is applied, the velocity stays constant. Since this friction compensation is basically a static friction–velocity map, it does not need many system resources. With the property that it is independent from the surrounding control algorithm, it fulfills all requirements (see Paragraph 4.5.2 and Table 4.4) for a friction compensation. It was utilized for all further
Figure 4.27: Validation of the exponential friction compensation: \(-\cdot\cdot\cdot\) desired angle; \(-\cdot\cdot\) real motor; \(-\cdot\cdot\) ideal motor.

Figure 4.28: Estimated friction torque.
experiments, if friction compensation was necessary.

<table>
<thead>
<tr>
<th>Requirement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear behavior</td>
<td>✓</td>
</tr>
<tr>
<td>Good behavior for small input signals</td>
<td>✓</td>
</tr>
<tr>
<td>Prevent chattering</td>
<td>✓</td>
</tr>
<tr>
<td>Assure back drivability</td>
<td>✓</td>
</tr>
<tr>
<td>Independent from the control algorithm</td>
<td>✓</td>
</tr>
<tr>
<td>Saving resources</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 4.4: Requirements for friction compensation.

4.5.6 Determination of the moment of inertia

The inertia $J$ had to be determined for the actuators, because the clamps for the links and the air bearings had to be included, i.e. the values given by the manufacturer could not be
used. A different inertia was especially expected for the shoulder actuator, since the shaft is fixed to the glass and its case is rotating.

A step input was applied to the P-controlled actuator with exponential friction compensation. Using \texttt{fminsearch} of the Matlab optimization toolbox, a simulated step response was fit to the measured one. Both curves are shown in Figure 4.30. The inertia was determined for the shoulder as $J_S = 3.41\, \text{kgm}^2$ and for the elbow as $J_E = 1.25\, \text{kgm}^2$.

Figure 4.30: Simulated step response matched to the real one: \(\cdots\cdot\cdots\cdot\) : step input; \(\cdots\) : real step response; \(\cdots\cdots\cdots\cdots\) : simulated step response.
Chapter 5

Control of the flexible manipulator

The control of flexible robots is a lot more complex than the control of rigid robots. The control objective in both cases is usually the positioning of the end-effector with respect to the base frame of the robot. This should be done as precisely as possible and often as fast as possible.

In the rigid case, a simple position or velocity control of the actuators satisfies the control objective, because it is assumed that there are no significant dynamics between the motor shaft and the behavior of the attached arm.

In the flexible case, each arm is a part of the manipulator with very complex dynamics. It can experience several modes of oscillation and has solid friction components. The actuators themselves have joint flexibility due to their construction. To meet the control objective, these additional dynamic effects have to be taken into account.

This chapter briefly describes two different control approaches and the experimental results of their implementation.

5.1 Passivity based controller

A passivity based controller is often used for applications where no plant model exists or the model is inaccurate. It is based on the idea of pulling the kinetic energy out of
the system to stabilize it. The viewpoint moves away from the idea of a system with internal states to a device that interacts with its environment by transforming inputs to outputs. Input/output pairs are called passive when the system between them dissipates energy, i.e. it could be modelled analog to an electrical system containing only resistors, inductors, and capacitors. If such an input/output pair can be found for the system, the passivity theory says that any passive controller in a negative feedback loop can stabilize the closed-loop system.

Damaren [4] proves the passivity for a two link flexible manipulator using the passive input/output pair: actuator torques/cartesian endpoint rates. Based on this finding, a passive controller can stabilize the manipulator with simultaneous vibration suppression.

5.1.1 Control law

A PD-control law proposed by Damaren [4] to stabilize a flexible two link manipulator is

\[
\tau = -J^T_\theta [K_d \dot{\rho}_\mu + K_p (\rho_\mu - \rho_d)]
\]

(5.1)

where \(\tau\) is the vector of actuator torques, \(\rho_\mu\) is the position vector of the end-effector in cartesian coordinates (\(\dot{\rho}_\mu\) is its time derivative), \(\rho_d\) is the desired position in cartesian coordinates, \(J_\theta\) is the Jacobian of the rigid two link manipulator, and \(K_p, K_d\) are the PD-gain matrices. The closed loop system can be expected to be stable for \(K_p = K_p^T\), \(\det(K_p^T) > 0\) and \(K_d = K_d^T\), \(\det(K_d^T) > 0\). The endpoint position \(\rho_\mu\) is generated by

\[
\rho_\mu = (1 - \mu)\rho_r + \mu\rho_f
\]

(5.2)

where \(\rho_r\) is the cartesian position assuming pure rigid links and \(\rho_f\) is the real end-effector position. The factor \(0 < \mu < 1\) assures that the elastic coordinates stay observable – it should be chosen close but not equal to one.
5.1.2 Endpoint position estimation

The end-effector position $\rho_f$ is determined by approximating the beam shape with a $n^{th}$ order polynomial $w(x)$. Link 1 is equipped with three strain gage bridges; together with the boundary conditions $w(0) = 0$ and $w'(0) = 0$ the five coefficients of a fourth order polynomial can be determined. Link 2 can be approximated by a third order polynomial, because it has only two strain measurement locations.

**Determination of deflection and angle of each flexible link**

Link 1 is approximated by

$$w_1(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4. \quad (5.3)$$

The two boundary conditions lead to $b_0 = 0$ and $b_1 = 0$. The strain at position $ij$ is approximated as

$$\epsilon(x_{ij}) = -\frac{t_i}{2}w''_i(x_{ij}) \quad (5.4)$$

$$w''_i(x) = 2b_2 + 6b_3x + 12b_4x^2 \quad (5.5)$$

where $t_i$ denotes the thickness of link $i$. The coefficients can be determined by an on-line solution of

$$\begin{bmatrix} b_2 \\ b_3 \\ b_1 \end{bmatrix} = -\frac{2}{t_i} \begin{bmatrix} 2 & 6x_{1A} & 12x_{1A}^2 \\ 2 & 6x_{1B} & 12x_{1B}^2 \\ 2 & 6x_{1C} & 12x_{1C}^2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_A \\ \epsilon_B \\ \epsilon_C \end{bmatrix}. \quad (5.6)$$

For Link 2 with

$$w_2(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \quad (5.7)$$

the two non-zero coefficients $a_2$ and $a_3$ are determined in a similar way.

With the coefficients calculated, the deflection $v_i$ and the angle $\alpha_i$ of the end of the link
is given by

\[ v_1 = b_2 L_1^2 + b_3 L_1^3 + b_4 L_1^4 \]  \hspace{2cm} (5.8)

\[ \alpha_1 = 2b_2 L_1 + 3b_3 L_1^2 + 4b_4 L_1^3 \]  \hspace{2cm} (5.9)

where \( L_1 \) denotes the length of the flexible part of Link 1. The deflection and the angle for Link 2 are calculated similarly.

**Flexible forward kinematics**

Flexible forward kinematics were determined to map joint angles, link deflections, and link angles to cartesian coordinates. They were derived geometrically. Since the deformations were small compared to the length of the links, it was assumed that the length of each link from the root to the end stays constant.

\[ \rho_{\mu} = \begin{bmatrix} x_f \\ y_f \end{bmatrix} \]  \hspace{2cm} (5.10)

\[ x_f = (l_{1a} + l_{1b}) \cos \theta_1 - v_1 \sin \theta_1 + l_{1c} \cos(\theta_1 + \alpha_1) \\
+ (l_{2a} + l_{2b}) \cos(\theta_1 + \alpha_1 + \theta_2) - v_2 \sin(\theta_1 + \alpha_1 + \theta_2) \\
+ l_{2c} \cos(\theta_1 + \alpha_1 + \theta_2 + \alpha_2) \]  \hspace{2cm} (5.11)

\[ y_f = (l_{1a} + l_{1b}) \sin \theta_1 + v_1 \cos \theta_1 + l_{1c} \sin(\theta_1 + \alpha_1) \\
+ (l_{2a} + l_{2b}) \sin(\theta_1 + \alpha_1 + \theta_2) + v_2 \cos(\theta_1 + \alpha_1 + \theta_2) \\
+ l_{2c} \sin(\theta_1 + \alpha_1 + \theta_2 + \alpha_2) \]  \hspace{2cm} (5.12)

\( l_{ik} \) denote the length of each section of the manipulator as shown in Figure 5.1.
5.1.3 Experimental results

The control law 5.1 was implemented with the parameters

\[
\mu = 0.6 \quad K_p = \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix} \quad K_d = \begin{bmatrix} 200 & 0 \\ 0 & 200 \end{bmatrix}.
\]

In order to get reasonable derivatives the measured signals were filtered by a second order digital Butterworth filter: The filter for the angles had a cutoff frequency of 10 Hz, and the filter of the strain measurements had a cutoff frequency of 50 Hz.

Generating the desired trajectory

The desired trajectory for the end-effector was designed to move the manipulator from rest at one position to rest at a second position. To keep the acceleration finite, the slope of the trajectory had to be zero at the start and end. With these four constraints \( x_i(0) = 0, \quad x'_i(0) = 0, \quad x_i(T) = e, \quad x'_i(T) = 0 \), the trajectory for each coordinate \( x_i \) was a third order
polynomial

\[ x_i(t) = c_{i0} + c_{i1}t + c_{i2}t^2 + c_{i3}t^3 \]  
(5.13)

\[
\begin{align*}
  c_{i0} &= 0 \\
  c_{i1} &= 0 \\
  c_{i2} &= \frac{3e_i}{T^2} \\
  c_{i3} &= -\frac{2e_i}{T^3}
\end{align*}
\]

where \( e_i \) is the end-position of the trajectory and \( T \) is the duration.

**Tracking and disturbance rejection**

The trajectory time \( T = 3s \) was chosen for a movement from \([x = 1.1m, y = 0.4m]\) to \([0.5m, 0.9m]\). The results in cartesian coordinates are shown in Figure 5.2 and 5.3. Two cases are shown in each figure:

1.) The controller is using the flexible feed-back \( \mu = 0.6 \)

2.) The controller is using the rigid feed-back \( \mu = 0 \).

A second experiment emphasizes the disturbance rejection of the controller (Figure 5.4) where a force pulse was applied to the end-effector.

The controller is able to move the end-effector along a desired trajectory with only small oscillations. But when the end-effector is approaching the desired position, the controller introduces a fast oscillation with small magnitude. This oscillation likely arises from the delay caused by the derivation and by the filtering of the signals. The comparison to the rigid feed-back case shows, that this oscillation is introduced by the flexible feed-back signal. Since the motor torques are chattering with the frequency of the oscillation, this behavior is not desirable, and the oscillation should be reduced in further approaches.

The reason for the tracking delay is the pure position control \( (\dot{\rho}_d = 0) \), i.e., providing the velocity trajectory in addition to the position trajectory will reduce this lag. The damping of disturbances is, as Figure 5.4 shows, quite fast.
Figure 5.2: Following x-trajectory:
- - · - : desired position; - : flexible feed-back; - - · - : rigid feed-back.

Figure 5.3: Following y-trajectory:
- - · - : desired position; - : flexible feed-back; - - · - : rigid feed-back.
5.2 Observer based controller

A model based controller for a one flexible link manipulator was developed by Ostojic [16]. The main property of this controller is that it does not require strain gage measurements of the flexible link. (It will be assumed for the investigation of this controller that no strain gage measurements are accessible). The control signal is generated by a recursive control law, and the states of the arm are provided by a fourth order Luenberger observer.

5.2.1 Model of one flexible link manipulator

A simple model was used to generate the observer, see Figure 5.5. The flexible link is replaced by a torsional spring between the motor shaft and a rigid massless link. The resulting model

\[
\begin{align*}
J_a \ddot{\theta}_a &= (\theta_m - \theta_a)k - b_a \dot{\theta}_a \\
J_m \ddot{\theta}_m &= \tau - b_m \dot{\theta}_m - (\theta_m - \theta_a)k
\end{align*}
\] (5.14)
Figure 5.5: One flexible link simplification (grey: real arm).

is a second order motor model combined with a second order spring-mass system. $\tau$ is the motor torque, $k$ is the spring constant, $J_m$ is the inertia of the motor, $J_a$ is the inertia of the arm, $b_{m,a}$ are the damping factors of motor and arm, and $\theta_{m,a}$ are the angles of motor and arm, respectively.

5.2.2 Control law

The control law has the recursive structure

$$
\tau(n+1) = \tau(n) + \gamma \sigma(n)
$$

$$
\tau(n+1) \in [\tau_{\text{min}}, \tau_{\text{max}}]
$$

(5.15)

where $n$ is the discrete time, $\gamma$ is a constant parameter, $\sigma(n)$ is the desired error-dynamics, and $[\tau_{\text{min}}, \tau_{\text{max}}]$ are the control signal limits. $\sigma(n)$ can be an arbitrary function, but it is recommended to use the same order differential equation than the system model. The function should provide a fast but suitable damped error-dynamics. For this plant

$$
e^{(4)} + 4a\ddot{e} + 6a^2 \dot{e} + 4a^3 e + a^4 e = 0 = \sigma(n)
$$

(5.16)
was used as error dynamics, where $e = \theta_d - \theta_a$ is the error, and $a$ specifies the location of the four poles. Figure 5.6 shows the asymptotic reduction of an initial error of 0.05\(\text{rad}\). Increasing $a$ leads to a faster error reduction, but it is upper bounded due to control signal limitations. Using Equation 5.16 and the model 5.14, the error-dynamics can be written as

$$\sigma(n) = a_0\theta_d - a_4\ddot{\theta}_m - a_3\dot{\theta}_m - a_2\ddot{\theta}_a - a_1\dot{\theta}_a - a_0\theta_a$$

(5.17)

\begin{align*}
a_0 &= a^4 \\
a_1 &= 4a^3 - \frac{4ak}{J_a} + \frac{kb_a}{J_a^2} \\
a_2 &= 6a^2 - \frac{4ab_a}{J_a} - \frac{k}{J_a} + \frac{b_a^2}{J_a^2} \\
a_3 &= \frac{k}{J_a} (4a - \frac{b_a}{J_a}) \\
a_4 &= \frac{k}{J_a}.
\end{align*}

For the choice of $\gamma$, Ostojic [16] proposes $\gamma = \frac{J_a J_m}{k}$. 

Figure 5.6: Error dynamics for recursive controller.
5.2.3 State observer

The feed-back information needed for the controller was not accessible directly from measurements, so a full-order Luenberger observer was utilized to provide the angle $\theta_a$, the rates $\dot{\theta}_m, \dot{\theta}_a$, and the accelerations $\ddot{\theta}_m, \ddot{\theta}_a$. The state-space model of the system is

$$\dot{x} = Ax + b\tau, \quad \theta_m = c^Tx$$

(5.18)

$$A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\frac{k}{J_m} & \frac{k}{J_m} & -\frac{b_m}{J_m} & 0 \\
\frac{k}{J_a} & -\frac{k}{J_a} & 0 & -\frac{b_a}{J_a}
\end{bmatrix}, \quad b = \begin{bmatrix}
0 \\
0 \\
\frac{1}{J_m} \\
0
\end{bmatrix}, \quad c = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}, \quad x = \begin{bmatrix}
\theta_m \\
\theta_a \\
\dot{\theta}_m \\
\dot{\theta}_a
\end{bmatrix}$$

(5.19)

The full-order Luenberger observer is shown schematically in Figure 5.7 and defined by

$$\dot{\hat{x}} = A\hat{x} + b\tau + K(\theta_m - \hat{\theta}_m)$$

$$K = [k_1 \quad k_2 \quad k_3 \quad k_4]^T.$$  

(5.20)

To damp the observer critically, the four observer poles are placed at $-p$, so the resulting observer gains are

$$k_1 = 4p - \frac{b_a}{J_a} - \frac{b_m}{J_m}$$

$$k_3 = 6p^2 - \frac{k + k_1b_a}{J_a} - \frac{k + k_1b_m}{J_m} - \frac{b_a b_m}{J_a J_m}$$

$$k_2 = \frac{J_m}{k} \left(4p^3 - \frac{k_3b_a + k_1k}{J_a} - \frac{k_1b_a b_m + k_b a + k b_m}{J_a J_m} \right)$$

$$k_4 = \frac{J_m}{k} \left( p^4 - \frac{k_3k}{J_a} - \frac{k_2k_b a + k_1k b_m}{J_a J_m} \right).$$
5.2.4 Experiments for one flexible link

For the experiments with only one flexible link, the shoulder motor had to be fixed. This was assured by mounting a rigid link between shoulder and elbow actuator and a cross beam to the wall of the facility. Since the air bearing is moving together with Link 2, it must be allowed to float during the experiments.

The parameters used for the experiments are collected in Table 5.1. They were either known or initially guessed and then adapted to improve the performance of the system. To demonstrate the full performance of the controller, two step inputs of $\pm \frac{\pi}{2}$ were applied to the system, once with an observer based controller for rigid arms and once with the observer based controller for flexible arms. The results can be seen in Figure 5.8, where the dashed line shows the desired angle, the dash-dot line shows the step response of the controller for rigid links, and the solid line shows the step response of the controller for flexible links. The observer based controller shows a far better performance than the controller for rigid links. The desired angle is reached with almost no overshoot, or in other words: The error
### Table 5.1: Parameters for control of flexible Link two.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td>Motor inertia</td>
<td>$J_m$</td>
<td>1.25</td>
<td>$kgm^2$</td>
</tr>
<tr>
<td>Arm inertia</td>
<td>$J_a$</td>
<td>3.5</td>
<td>$kgm^2$</td>
</tr>
<tr>
<td>Motor friction coeff.</td>
<td>$b_m$</td>
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<td>$\frac{Nms}{rad}$</td>
</tr>
<tr>
<td>Arm friction coeff.</td>
<td>$b_a$</td>
<td>0</td>
<td>$\frac{Nms}{rad}$</td>
</tr>
<tr>
<td>Spring constant</td>
<td>$k$</td>
<td>200</td>
<td>$\frac{Nm}{rad}$</td>
</tr>
<tr>
<td>Error-dynamics pole</td>
<td>$a$</td>
<td>$-10$</td>
<td></td>
</tr>
<tr>
<td>Observer pole</td>
<td>$p$</td>
<td>$-30$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.8: One flexible link observer based angle control: - - : desired; – - : rigid controller; – : flexible controller.
is smaller than 2% after 1.15s. It should be noted, that the weight of the end-effector is 8.1 kg.

### 5.2.5 Extension to two links - experiments

Since the results for one link were very good, a simple extension to the two link case was made: An independent joint control was implemented by utilizing the same controller also for the shoulder motor but with different parameters. The parameters were found during experiments without the end-effector mounted, see table 5.2. With these parameters a step was applied to both actuators. The step responses are compared to those applied to the joints with simple PD control. Figures 5.9 and 5.10 show the step responses, where the dash-dot line shows the response with PD control, and the solid line shows the response with observer based control.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
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<td>Motor inertia</td>
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</tr>
<tr>
<td>Arm friction coeff.</td>
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<td>$\frac{Nms}{rad}$</td>
</tr>
<tr>
<td>Spring constant</td>
<td>$k$</td>
<td>400</td>
<td>$\frac{Nm}{rad}$</td>
</tr>
<tr>
<td>Error-dynamics pole</td>
<td>$a$</td>
<td>$-10$</td>
<td></td>
</tr>
<tr>
<td>Observer pole</td>
<td>$p$</td>
<td>$-30$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Parameters for control of flexible Link one.
Figure 5.9: Two flexible link observer based angle control (Shoulder):—- desired; — — PD controller; — observer based controller.

Figure 5.10: Two flexible link observer based angle control (Elbow): —- desired; — — PD controller; — observer based controller.
Chapter 6

Conclusions and future work

The Watflex facility provides a good example for a 2-D space environment. The flexible manipulator mimics many effects that occur in real space applications. The two most important effects – friction in harmonic drives and flexible links – were addressed by this project.

Several friction phenomena were investigated, and some common friction models were presented. These models were utilized for friction compensation in different configurations. Experiments on the harmonic drive actuators showed that the static, compared to dynamic, friction effects are dominant. Therefore, a static model was finally chosen for friction compensation. Because of the high static internal friction of the actuators, the qualified friction compensation scheme was developed. The application of this friction compensation to the Watflex joints removed the non-linear behavior almost completely, so the friction compensated joints can easily be modelled.

With this friction compensated actuators given, two control approaches were applied: The passivity-based controller showed stable behavior, but it introduced a high frequency oscillation of the motor torques. The oscillation likely arises from the delay introduced by the filtering and by the derivation of the measured signals. The observer-based controller showed very good performance for one flexible link – it reduced the first mode of oscillation to a magnitude close to zero. Even for the two link case,
where the controller was applied as independent joint control, the reduction of oscillations had been quite high. This was remarkable, because the model for the observer was quite different to the real plant.

Future work on the friction compensation could be an adaptive approach to cover slowly changing parameters of the actuators and temperature effects. A dynamic friction model can be considered, but it should be balanced between the amount of increased performance and the disadvantages of additional states in the motor model.

The passivity based controller can definitely be improved. It is probably only a small step missing to a perfect working controller. Focus should be kept on the filtering and the derivation of the measured signals. To avoid the derivation of the angle measurement, a tachometer signal could be used, but usually these signals have to be filtered as well because of noise on them. Another, more sophisticated model for the beam shape could also be used to determine the endpoint position more accurate.

The observer based controller can be improved by modelling the two flexible link case for generating the observer. With an accurate state observation, the controller will be capable to stop the vibrations of the arms very efficiently and without strain measurements – as it was shown for one flexible link.

The recursive control law used for the observer based controller could also be implemented for using the real bending measurements. But this will need the first and the second derivative of the strain measurements, which will likely need a filter for the signals and cause delays.

Future experiments should be undertaken with both of the investigated control schemes, because there is a lot potential to increase their performance. Other control schemes should be investigated as well, because the findings are not only interesting for space applications but also very important for the growing number of lightweight robots used in industry.
Appendix A

A.1 Strain gage calibration in opposite direction

The calibration measurements described in 3.3 are also done in the opposite direction, i.e. the link was clamped at its end (near bridge 1C) and the force was applied near bridge 1A. The measurement setup for link two was equivalent.

Figure A.1: Static map of strain gage calibration for link one, clamped at 1C (○: bridge A; +: bridge B; *: bridge C; -: according regression).
Figure A.2: Absolute variance from linear regression of link one.

Figure A.3: Static map of strain gage calibration for link two, clamped at end (○: bridge A; +: bridge B; −: according regression).
Figure A.4: Absolute variance from linear regression of link two.
A.2 Friction compensation experiments for the elbow actuator

The properties of the elbow actuator are shown in Table A.1. The parameters for the exponential friction model are shown in Table A.2. Figures A.6 to A.7 show the measured friction–velocity maps.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>RFS-25-6018</td>
</tr>
<tr>
<td>Rated Output Torque</td>
<td>30Nm</td>
</tr>
<tr>
<td>Rated Output Speed</td>
<td>60rpm</td>
</tr>
<tr>
<td>Maximum Output Torque</td>
<td>100Nm</td>
</tr>
<tr>
<td>Torque Constant</td>
<td>$11.5 \frac{N m}{A}$</td>
</tr>
<tr>
<td>Inertia at Output Shaft</td>
<td>1.1kgm$^2$</td>
</tr>
<tr>
<td>Gear Ratio</td>
<td>1 : 50</td>
</tr>
<tr>
<td>Mass</td>
<td>6.8kg</td>
</tr>
</tbody>
</table>

Table A.1: Technical specifications of the shoulder motor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value for positive part</th>
<th>Value for negative part</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>$Nm$</td>
<td>4.9589</td>
<td>5.4392</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>$Nm$</td>
<td>0.5232</td>
<td>0.5556</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$\frac{N m s}{rad}$</td>
<td>1.2563</td>
<td>0.8162</td>
</tr>
<tr>
<td>$v_S$</td>
<td>$\frac{rad}{\pi}$</td>
<td>0.0402</td>
<td>0.0208</td>
</tr>
</tbody>
</table>

Table A.2: Static parameters of the exponential model for the elbow actuator.
Figure A.5: Static friction–velocity map of the elbow actuator.

Figure A.6: Positive part enlarged.
Figure A.7: Negative part enlarged.
Bibliography


