Volumetric Off-Road Tire Model for Electric Vehicle Application

Willem Petersen, ID # 20190824

Supervisor: Prof. John McPhee

University of Waterloo
Faculty of Engineering
Department of Systems Design Engineering

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Abstract

In recent years, electric vehicle and hybrid electric vehicle designs have assumed a prominent role in the business strategies of most automotive companies. With more and more companies developing vehicles partially or entirely powered by electric motors, the demand for efficient simulation tools including modelling capabilities of all major drive train components is increasing. For faster simulations, the vehicle dynamics models normally used in electric power train simulations are simplified to only represent major energy losses due to aerodynamic drag or rolling resistance. For the design process of planetary rovers, however, the low quantities and the fact that their power demand and consumption highly depends on the vehicle dynamics, more accurate models are necessary to meet the requirements. This report proposes a procedure for modelling planetary rovers including the crucial drive train component models and a novel off-road tire model. Thereby, linear graph theory and symbolic computing is used to assure an efficient simulation process.

Many symbolic and numeric formulations have been proposed for generating the equations of motion of multibody systems. A linear graph formulation procedure is chosen for this work because of its ability to model multi-domain systems. The linear graph-theoretic approach has proven to handle one-dimensional electrical components in the same framework as three-dimensional mechanical components. Furthermore, it has proven to formulate efficient sets of the governing equations in symbolic form providing the user the flexibility of interactive coordinate selection.

Using linear graph theory and its advantages, the primary goal is to develop an efficient modelling and simulation tool for planetary rovers which has the potential to be used for other applications of electric off-road vehicles. To ensure the validity of the subsystems models and the overall models generated, experiments using the facilities of the Canadian Space Agency will be performed to obtain realistic data sets to compare against the simulation results. Moreover, the modelling tool will be validated by comparing simulation results from other well established simulation software packages.
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Introduction

Electric vehicle (EV) modelling and simulation tools are of growing interest to the automotive industry as the demand for alternative energy propulsion systems in commercial vehicles is increasing. These design tools generally support the modelling of electric drive train components on a system level, where the subsystem models are based on experimental data in the form of numeric look-up tables. Normally, the vehicle and especially the tire dynamics are poorly represented and the hierarchical configuration of such block diagram based models results in a fixed causality. The objective of this project is to develop a modelling and simulation facility for electric vehicles, and in particular planetary rover applications, with a focus on dynamics of off-road tires and vehicles. Using graph theory and symbolic computing, the developed formulation will automatically generate multibody multi-domain models of planetary rovers including the crucial drive train component models and a novel off-road tire model.

1.1 Background

Several modelling and simulation platforms for electric vehicles or even hybrid electric vehicles (HEV) have been developed since the early 1990s. Advanced Vehicle Simulator (ADVISOR) and Powertrain System Analysis Toolkit (PSAT) rank among the most famous ones. These design tools allow the modelling and simulation of hybrid electric vehicles with a focus on the power and propulsion system. Generally, these software packages are designed to yield relative results between different drive train configurations to avoid advanced and possibly solver-costly models of vehicle and tire dynamics. For planetary rover design however, it
is important to accurately simulate the dynamics of the vehicle. Since rovers are designed to move in various planetary environments (Earth, Moon and Mars), the vehicle dynamics models require accurate representation of tire/soil interaction for a variety of wheel and terrain types including hard rock or soft sand, and rubber or metal wheels.

For this interaction model, a volumetric approach will be used which was developed by Gonthier and McPhee and has been validated for various robotic tasks. In cooperation with the Canadian Space Agency (CSA), further development has been established on the volumetric-type contact model which is of the same continuous structure as Hertz-type models. The key difference between the two methods is the different interpretation of contact stiffness and that the volumetric contact model takes into account the pressure distribution over the contact area. The volumetric properties can be derived directly from the shape and the position of the objects in contact. Depending on the geometry and the deformed states of the interacting objects, the calculation of these volumetric properties can be very extensive, which will be supported by Parallel Geometry Inc. (LLG), whose parallelized algorithm will provide the needed tool for the calculation. Using the software supplied by LLG and high-performance computers based on parallel architectures, the volumetric approach is expected to be much faster than other high-fidelity models of tire/soil interaction based on finite and/or distinct elements. However, the volumetric model must be validated experimentally for which the CSA team will provide its expertise and planetary rover test facilities. This project combines the teams of the University of Waterloo (UW), the CSA, and LLG, all of which have worked together in the past.

1.2 Motivation and Challenges

There are several reasons for developing a modelling and simulation facility for electric vehicles and planetary rovers. First, as it is true for the design phase of any system, modelling and simulating planetary rovers cuts down on time and cost required for prototyping such vehicles. In fact, the modelling and simulation process is indispensable in the design process of vehicles. Secondly, it is nearly impossible to test prototypes of planetary rovers in a realistic environment as they are designed to move on the Moon or even Mars. This is also a reason why the modelling has to be as accurate as possible. Many commercially available software offer the simulation of either the drive train or the vehicle dynamics, but usually lack in the representation of the other components. To obtain realistic results, it is necessary to combine accurate models of each system and simulate them in one overall electric vehicle
model and not independently from each other. To achieve faster simulations, the models will be generated in a symbolic computing environment. The subsystems of the drive train are often modelled using experimental data, which has to be included into the symbolic models. To assure accurate calculations of the vehicle trajectories, the vehicle dynamics require the consideration of the terrain, which is often soft and/or uneven. Therefore, an accurate off-road tire model is a key to the proper prediction of the vehicle dynamics. A novel approach for the vertical force calculation will be implemented to overcome problems of many tire models with multiple contacts and contacts with sharp edges. For all of the discussed models, it should be assured that the experiments to achieve model parameters or required data sets are within a reasonable scope.

Although widely used in modelling hybrid electric drive train components, numeric look-up tables have not been used in symbolic modelling of hybrid electric vehicles. The difficulty of such an implementation is to generalize the procedure of generating sufficient functions by curve-fitted numeric data of different levels of complexity, e.g. motor curves, efficiency maps or highly nonlinear characteristic curves of batteries. In addition, the data can be of discontinuous nature due to operation limits given by certain components within the subsystem. The biggest challenge in this project, however, will be the modelling of the off-road tire model. In particular, the contact problem between two soft objects, the tire and the soft soil, is the reason for this major difficulty. Due to the fact that each body deforms during contact, the penetration geometry can not simply be calculated by comparing the position of the interacting objects. The deformed shape of each body has to be considered which is dependent on the contact force. This creates an implicit problem which makes the use of conventional tire models inadequate. Several two-dimensional approaches have been suggested to achieve analytical models of such tire/soil interaction problems. However, for the implementation of the volumetric contact model, these planar approaches have to be extended to 3D for which the degree of difficulty increases with the complexity of the assumed tire geometry, e.g. cylinder, torus or a realistic CAD model. Since the calculation of the volumetric properties of the penetration geometry can be computationally expensive, the efficient determination of said properties using provided parallel algorithms and high-performance hardware will be a challenge by itself.
1.3 Application

The obvious field of application of the proposed research is to design planetary rovers which operate on the Moon or Mars. The Mars exploration program “Mars Science Laboratory — The Next Mission to Mars” is one of the biggest projects run by NASA in close cooperation with the CSA, and future missions are already in planning. However, this research is also beneficial in designing electric vehicles of all kinds with possible applications in off-road operation, e.g. agricultural vehicles and off-road trucks. Although driven by the CSA for the use in rover simulation, the proposed modelling facility can be easily used to model electric vehicles as they are on the agenda of all major car companies. In addition, implementation into the Maple programing language will allow for the subsystem models and also the proposed modelling approaches to be applied in many different fields of multibody multi-domain simulations.
2

Literature Review

2.1 Analytical Multibody Systems Modelling

The first textbook on mechanical multibody dynamics was written by Wittenburg (1977) starting with the kinematics and dynamics of rigid bodies. A textbook published by Schiehlen (1986) presents multibody systems, finite element systems and continuous systems as equivalent models for mechanical systems. The computer-aided analysis of multibody systems was considered in 1988 by Nikravesh (1988) for the first time. Basic methods of computer aided kinematics and dynamics of mechanical systems are shown by Haug (1989) for planar and spatial systems. In his first textbook, Shabana (1989) deals in particular with flexible multibody systems, whereas he concentrates on computational dynamics of rigid multibody systems in his second book (Shabana 1994). Kecskemethy and Hiller (1994) introduced an object-oriented formulation to model the dynamics of multibody systems. Contact problems in multibody dynamics were highlighted by Pfeiffer and Glocker (1977). The symbolic modelling approach of multibody systems was described in the book of Samin and Fisette (2003), while the application of multibody formulations to vehicle dynamics was discussed in the textbook by Blundell and Harty (2004).

As the demand for multi-disciplinary modelling and simulation tools increases, especially in the area of automotive engineering and electric vehicle design, the development of formulations that automatically generate the governing equations of such systems has become a research field of growing importance. Such formulations create models of the overall system given the constitutive models of system components, or even entire subsystems, and a representation of the system typology (Samin and Fisette 2003). In this project, linear
graph theory is used to represent the system topology and symbolic computing is exploited to generate the system equations.

Linear graphs directly reflect the system topology and, due to their domain independence, the graphs can be easily applied to hybrid systems consisting of components from multiple domains, e.g. mechanical, electric, hydraulic (Sass et al. 2004; McPhee et al. 2004). The unique topology representation with linear graphs allows to conveniently model 3D mechanical system (McPhee et al. 2004), which can be a tedious process using other graph-based modelling approaches (Samin et al. 2007). Also, the linear graph representation of the system allows for easy generalized coordinate selection as shown in the work of McPhee and Redmond (2006); McPhee (1997).

Linear graph theory provides the theoretical foundation to this project as it has a long history in the modelling and simulation of engineering systems (Chandrashekar, Roe and Savage 1992). Leonard Euler is widely regarded as the originator of graph theory (Hopkins and Wilson 1997), as his article about a particular bridge connectivity problem in Königsberg from 1736 is of considerable importance to the development of graph theory (Biggs et al. 1998). Since then, linear graph theory has gone through enormous development driven by a wide variety of applications. Initially developed to model electrical systems (Seshu and Reed 1961), linear graph theory was first applied to a general group of lumped-parameter models in electrical and mechanical domains by Koenig and Blackwell (1960), which is based on the work of Trent (1955).

Andrews and Kesavan (1973b) introduced a “vector-network model”, which was used by Baciu and Kesavan (1995) to model the interaction between multiple rigid bodies in 3D space. McPhee generalized this work to model planar (McPhee 1998) and three-dimensional multibody systems using absolute and joint coordinates (McPhee 1996), which was extended for the use of indirect coordinates by McPhee and Redmond (2006). Savage (1997) has shown the advantage of subsystem models when modelling large systems using linear graph theory. Furthermore, Shi, McPhee and Heppler (2001) have shown that the concept of virtual work can also be included in graph-theoretic formulations that even allow the user to choose a desired set of coordinates.

The development in these modelling methods for either electrical or mechanical systems consequently lead to the combination of the two domains, and Muegge (1996) was the one to first model planar electromechanical systems using linear graph theory. This research was further developed and generalized for 3D systems by Scherrer and McPhee (2003). Ex-
amples for modelling multi-physical systems are shown in the work of Sass et al. (2004) and Samin et al. (2007). In these approaches, different formulations were used to model 3D mechanical systems and 1D electrical lumped-parameter systems. The method suggested by Schmitke and McPhee (2005) allowed to model complex multibody multi-domain systems using the same formulation based on linear graph theory and the principle of orthogonality.

The simulation of many tasks in multibody dynamics require the modelling of contacts, which has been done in a number of projects within the Motion Research Group at UW. Hirschkorn et al. (2006) have modelled a piano hammer mechanism in which the contact was treated as coupled applied forces. Another example of contact modelling using linear graph theory is given in the work of Millard, McPhee and Kubica (2008) in which a volumetric contact model was chosen to represent the soft-hard contact between the foot tissue and the hard floor. Finally, the ability of graph theory to be used in modelling the interaction between a tire and the road can be reviewed in Schmitke et al. (2008).

### 2.2 Electric Vehicle Modelling

Along with the growing demand of electric vehicle research, modelling and simulation software for such vehicles have been developing quickly. The software packages ADVISOR and PSAT belong to some of the most known simulation tools. In both cases, the software provides a number of subsystem models of the drive train components, which the user can combine in different ways to create different drive train configurations. However, such software packages usually lack good vehicle dynamics models due to the fact that high-fidelity vehicle models require significant computational time. Therefore, the purpose of programs like ADVISOR and PSAT is more to compare different drive train configurations based on relative performances than to simulate real overall vehicle performances.

ADVISOR was first developed in 1994 at the National Renewable Energy Laboratory in cooperation with the U.S. Department of Energy (DOE). Its primary role is to model the interaction of hybrid and electric vehicle components and their impacts on the vehicle performance and fuel economy (Markel et al. 2002). ADVISOR is a purely empirical modelling program, i.e. the subsystem models rely on experimental data, and it was written in MATLAB/Simulink software environment (Wipke et al. 1999). Later, the PSAT software package replaced ADVISOR as the major hybrid electric vehicle evaluation tool of the DOE. PSAT has been developed by the Argonne National Laboratory (ANL) which is managed
by the University of Chicago and funded by the DOE. PSAT operates in a similar way as ADVISOR; however, it has advantages in modelling and especially in simulation capabilities over its predecessor. PSAT is used by the DOE and major automotive companies to develop control strategies (Rousseau et al. 2007a) and then to optimize these strategies (Cao et al. 2007; Johnson et al. 2000). The major drawbacks of these software packages are the lack of proper vehicle dynamics representation and the fact that the modelling process is of procedural nature. This means that the causality of each subsystem and the overall system model is fixed (Sinha et al. 2001). In causal models, procedures, e.g. models of subsystems or primitive models such as integrators, multipliers etc., determine dependent variables as functions of independent variables. These functions have to be evaluated in order, which requires the user to explicitly direct the solution flow of equations within a system model. Acausal models, on the other hand, are defined by a set of equations relating the time and the state variables to their time derivatives. The advantage is that the user does not have to define the causality of these equations.

Besides the advantage of a graphical user interface (GUI), ADVISOR and PSAT include subsystem models of each drive train component relevant for hybrid electric vehicle designs. These subsystem models include motors/generators, IC engines, transmissions, and clutches. Models of most commonly used high-performance Lithium-ion batteries (Axsen et al. 2001) have been part of an ongoing research at the ANL (Nelson et al. 2002a). These battery models have been developed by Nelson et al. (2002b) to simulate the performance and energy storage systems of plug-in hybrid electric vehicles (PHEV) and fuel cell vehicles (Nelson et al. 2007). These battery models are included in PSAT and have been used by Sharer et al. (2006) to determine battery requirements and by Rousseau et al. (2007b) to evaluate battery prototypes. Since these models are dependent on experimental data, characteristic battery curves, all of the above models include test results of batteries that were achieved following the Battery Test Manual developed by the Partnership for a New Generation Vehicle (2001).

For more flexibility, researchers have developed their own drive train evaluation tools such as V-Elph by Butler et al. (1999), which is also a system-level modelling and simulation tool and was further developed by Rahman et al. (2000). However, using the advantages of graph-theoretic modelling and symbolic computing, Vogt et al. (2008) have developed an acausal electric vehicle model, including four electric in-wheel motors, that was simulated in real-time. The in-wheel motor models are based on electric motor models that were formerly developed using linear graph theory by Schmitke and McPhee (2003) and have also been used in Schmitke (2004) and McPhee (2005). Further development of symbolic acausal models of
hybrid electric drive train subsystems is a subject of ongoing research in the Motion Research Group to create a simulation facility for various types of electric vehicles for road and off-road application.

2.3 Tire Modelling

As mentioned earlier, the vehicle dynamics are overly simplified in most electric vehicle modelling and simulation software packages and the tire dynamics are often only represented through a simple rolling resistance force applied to the chassis (Rousseau et al. 2004). Therefore, if the goal is it to simulate the overall performance of an electric vehicle, one has to include a vehicle model of higher fidelity that properly represents the vehicle dynamics including tire dynamics. In the case of off-road electric vehicles such as planetary rovers, the tire model requires consideration of dynamics of the soft soil on which they are rolling.

Possible modelling methods for tires can be of numerical, empirical or purely symbolic nature. Numerical models such as finite element tire models are models of very high fidelity (as in the work of Fervers (2004)) but often computationally too expensive for multi-body vehicle simulations. Empirical models can be of symbolic nature; these models require experimentally gained data, which is used to curve fit functions for tire parameters as in the case of the Pacejka tire model by Pacejka (2006). The data collection can be tedious and often requires expensive equipment such as tire test benches. Therefore, a purely symbolic model with fewer parameters such as the Fiala tire model (also used in the work of Morency (2007)) is often the best choice for vehicle dynamics simulations. However, when it comes to uneven and soft terrain, the tire model needs to be extended to include the dynamics of the compliant ground. Moreover, the transverse tire dynamics change depending on the compound of the soil. A review of such tire models including the interaction with soft soil is given in Wong (1991).

For tires on hard surfaces, Sayers and Han (1996) listed a number of tasks that need to be completed to properly calculate the tire forces and moments that can be applied to the multibody system model of the vehicle:

1. Define acting location of tire force
2. Determine vertical force
3. Define local tire coordinates
4. Determine tire kinematics

Following these tasks, the tire forces can be calculated with the tire model. Using the following equations (Schmitke et al. 2008), those calculated tire forces and moments can then be applied to the desired frame on the multibody system:

\[
F_C = F_P \quad (2.1)
\]
\[
M_C = M_P + r_{P/C} \times F_P \quad (2.2)
\]

where \(F_C\) and \(M_C\) are the forces and moments applied to the wheel center respectively, \(F_P\) and \(M_P\) are the forces and moments acting in the contact patch respectively and \(r_{P/C}\) is the vector between the two locations. Schmitke, Morency and McPhee (2008) have realized the implementation of these tasks into a symbolic environment, DynaFlexPro. To extend the implementation of tire models for off-road application capabilities, a model for the soft soil has to be considered before proceeding with the tasks listed above.

These soil models inherit, similar to the tire model, different levels of fidelity depending on the approach. These approaches include finite element models, granular models using distinct element methods (DEM), and various continuous models with hyperelastic and hyper-viscoelastic properties. Yong and Fattah (1967) first used finite elements to determine the soil deformation under a rigid wheel. This model was further developed by Boosinsuk and Yong (1984), and the most recent finite element models of soil/tire interaction include a deformable tire and inflation pressure distribution as in the work by Fervers (2004). A similar approach to FE models is the distinct element method. In the DEM approach, the compliant ground is modelled with small granular soil elements, each of which interacts with its adjacent elements. Most recent work has been published by Khot et al. (2007) and by Nakashima and Oida (2004), who combined a FE model of the soil with a top layer of distinct elements that then directly interact with the tire.

For multibody systems, the most commonly used model to represent the soil in a tire/soil interaction model is based on the pressure distribution by Bekker (1962). Bekker’s model is a continuous model with hyperelastic properties for the vertical compaction of the soil (Bekker 1960). This soil model includes three Bekker parameters that can be obtained from experiments with a bevameter or a simple cone penetrometer (Bekker 1969). The Bekker model has been improved and extended by contributions of many researchers such as Reece (1964) who suggested a similar model with slightly different parameters. Later, Wong (1993) introduced different models for different sections over the tire in which the load on the soil can
be different. Further developments were made by others to include the effects of viscoelasticity (Grahn 1992) or soil deposition (Krenn and Hirzinger 2008b, a).

Considering the previously listed task suggested by Sayers and Han (1996), the Bekker model is used to calculate the normal force which is then applied to the tire model to calculate the tire forces and moments. In many applications of off-road vehicles, the tire is significantly stiffer than the soil. Therefore, the tire is often assumed to be rigid which simplifies the tire/soil interaction model to be only dependent on the soil dynamics and the tire geometry. Thus, the contact behaves like a hyperelastic spring, where the spring stiffness and the exponent representing the hyperelasticity are defined by Bekker’s soil parameters. Early contributions to rigid tire models were made by Bekker himself and Janosi (1961). By improving the expressions of the tire kinematics used in the model, Onafeko and Reece (1967) extended the previous work of Janosi and Hanamoto. That the assumption of a rigid tire is sufficient enough in many cases is also shown by Wong (1993) who distinguished between towed (Wong and Reece 1967) and driven wheels (Wong 1991). In fact, prototypes of planetary rovers are often designed to run on aluminum or even titanium tires for which this assumption of rigid tire properties has been proven to be valid by Bauer et al. (2005a). This is also shown in their extended work (Bauer et al. 2005b).

Early work on contact models including deformable tires and Bekker soil has also been done by Janosi and Hanamoto (1961). A summary of 10 years of research on vehicle and terrain interaction is given by Schmid (1995), including several analytical soil contact models. For the normal force calculation in these models, the interpenetration area of a thin disc in contact with the deformed soil profile is determined by using different approaches (Schimd and Ludewig 1991). In those models the penetration area is converted into a representative tire deflection so that conventional spring tire model can be used to calculate the normal force. Harnisch et al. (2005) used one of Schmid’s approaches to develop the commercially available tire model @AS2TM for the implementation in ORSIS (Harnisch and Lach 2002), a simulation platform for heavy military trucks (Harnisch et al. 2003). Finally, Abd El-Gawwad et al. (1999d) implemented previously discussed continuous soil models into a multi spoke tire to calculate the normal force.

Whereas the FE models calculate the total force vector directly, the tangential forces in the symbolic modelling approach have to be calculated separately from the normal force (Sayers and Han 1996). In that case, the longitudinal force which defines the traction on the tire and the lateral force which defines the transverse dynamics of the vehicle are often calculated using a similar approach based on the horizontal soil strength. Therefore, Janosi and
Hanamoto curve fitted the shear tension-displacements relations that were obtained experimentally. They used exponential functions that start from zero, increase rapidly and approach a maximum shear stress value. The maximum stress can be mathematically described using Mohr-Coulomb shear failure criteria (Janosi and Hanamoto 1961). This approach has been investigated and especially improved for lateral forces by Schwanghart (1968).

The Janosi and Hanamoto approach works fine on most sands, saturated clay and fresh snow. An alternative description of the relation between deformation and shear stress was published by Wong (1993). This approach was developed to include soils in which the stress-displacement curve reaches a maximum shear stress and decreases again after the “shear-off” of the surface mat is initiated (Wong 1993). By relating the shear displacement of the soil in the tire contact patch to the tire kinematics, i.e. longitudinal slip and slip angle, one can calculate the longitudinal and the lateral force respectively. Grecenko (1967) suggested an alternative kinematic relation that can be used for such calculations. Recent developments in predicting the traction force of a heavy off-road vehicle has been performed by Li and Sandu (2007). In addition, Schreiber and Kutzbach (2007) have investigated zero-slip effects of off-road tire and proposed methods to incorporate these into traction force models. Due to the sinkage of the wheel into the ground and the resultant enlarged contact patch, there is still slipping between the wheel and the soil when the zero slip state used in conventional on-road tire models is reached (Schreiber and Kutzbach 2007). Since the same soil stress relation is used to model the lateral and longitudinal tire forces, Crolla and El-Razaz (1987) have suggested a method to combine the calculation of these forces and Li and Sandu (2008) further developed this idea.

In their series on tire/soil interaction modelling, Abd El-Gawwad, Crolla and El-Sayed (1999a) discuss modelling methods and the effect of straight (Abd El-Gawwad et al. 1999a) as well as angled lugs (Abd El-Gawwad et al. 1999c). Additionally, Abd El-Gawwad et al. (1999b) investigate the effect of camber on the tire performance.

A common problem of analytical off-road tire models is that they can not handle multiple contacts. Also, they lack an accurate prediction of the tire forces when they are in contact with sharp edges (Harnisch 2005). However, these are likely scenarios for a planetary rover considering the surface of the Moon or Mars. The volumetric contact model, which will be used for the normal contact force calculation of the off-road tire model proposed in this project, has been proven valid for these type of contacts.
2.4 Volumetric Contact Modelling

As Gilardi and Sharf mentioned in their literature survey on contact dynamics modelling, two different approaches can be chosen to model the contact between two objects. One approach is based on discrete or impulse-momentum methods, whereas the other approach is referred to as the continuous or force-based method. Since the former is generally used to model impact in which the contact time is very brief and rapid energy dissipation due to high accelerations and decelerations occur, continuous methods are normally used to model the contact between tires and roads or even soft soils. These continuous methods, also referred to as regularized or compliant contact models, can include models of Hertz, Hunt-Crossley or simple linear spring-dashpots (Gilardi and Sharf 2002); however, a contact detection algorithm has to be implemented for the application in tire modelling.

Unlike the previously stated compliant contact models, which calculate the normal contact force based on the penetration depth, the in this project proposed tire model determines contact forces based on volumetric properties of the contact problem. This contact model has been recently developed by Gonthier et al. (2005) and has proven to be valid for various examples of modelling contacts between two hard objects (Gonthier 2007). The required volumetric properties of the interpenetration volume of the given contact are (see Figure 2.1):

- penetration volume $V$,
- center of mass $CoM$,
- interia tensor $J$.

The volume metrics are obtained either analytically or numerically depending on the geometries of the object that are in contact (Gonthier et al. 2008a). The volumetric contact model is based on a Winkler foundation model as proposed by Johnson (1985), and one of the big advantages of this approach is that it can handle multiple contacts and contacts with sharp edges Gonthier (2007). Furthermore, it automatically leads to the consistent selection
of the point of action and the calculation of the rolling resistance moment (Gonthier et al. 2008b).

There already exists a precedent for a volumetric based tire contact model, in the modelling of tires on hard 3D roads used by the MSC.Adams software package. In the “3D Equivalent-Volume Road Model”, MSC.ADAMS computes an approximate volume of intersection between a tire and a 3D surface represented by triangular patches. However, instead of using the volume metrics directly, the volume is converted into an equivalent depth of penetration using empirical look-up tables. The penetration, which represents the tire deflection, is then used to calculate the normal load by considering a linear spring-damper tire model (MSC.Software Corp. 2005).
Progress to Date

Two different models are developed both with relevant content for the proposed research: a semi-symbolic model of a hybrid electric drive train and an off-road tire model. The hybrid electric vehicle model includes models of major drive train components that are part of a planetary rover design, and the off-road tire model defines the only interface between the vehicle model and the ground which is a big unknown for planetary environments.

For the hybrid electric vehicle (HEV) model, an architecture of a series hybrid was chosen, where the internal combustion (IC) engine is connected to the generator only. This results in a series setup of the drivetrain components in which the IC engine/generator assembly, also referred to as genset unit, provides electric power to the vehicle system, whereas a parallel setup would also provides mechanical power to directly drive the vehicle. The electric motor provides the tractive effort to the vehicle during acceleration. Furthermore, the generative capabilities of the electric motor are used during the deceleration of the vehicle (i.e. regenerative braking) which provides power to the system that would otherwise be lost.

One of the challenges in modelling the mechanics of an off-road tire is that the ground can not be expected to be rigid. A rigid road is a common assumption for the road in models of conventional tires. The soil dynamics have to be considered in the calculation of the forces in the soil/tire interface as they highly influence the dynamics and boundary conditions of the contact problem. Therefore, a conventional tire model has to be extended by an appropriate soil model to assure the validity of such models.
3.1 Empirical Electric Vehicle Model in Simulink

The HEV model components are created using MATLAB/Simulink, Maple11 and its multi-body simulation toolbox DynaFlexPro. These components are combined into a full vehicle model using Simulink which allows for easy connection between the different subsystems. The model is assumed to be a series hybrid, where the IC engine is only connected to the generator to provide electric power to the vehicle system, as opposed to a parallel hybrid, where the IC engine can be connected to the vehicle drivetrain. An illustration of the series HEV architecture with the modelled drivetrain components is shown in Figure 3.1 below.

![Series Architecture of HEV Model](image)

Figure 3.1: Series architecture of HEV model

and vehicle components are illustrated by the corresponding model block, which are fully parametrized and easily changeable to simulate different vehicle configurations. However, the semi-symbolic HEV model relies on test data for most of the vehicle components which include the drive cycle, the power control logic, the IC engine/generator setup (genset), the battery and auxiliary loads, the electric motor/generator, the transmission and the vehicle dynamics.

3.1.1 Modelled Electric Vehicle Components

The power control logic calculates the required power to propel the system based on the actual and desired speed of the vehicle and other possible resistances that need to be overcome. The
controller then defines the states and the required performance of each of the drive train components which are described in this subsection along with the other illustrated blocks (see Figure 3.1).

**Drive Cycle**

In order to test the performance and the efficiency of the chosen vehicle configuration, a number of predefined driving scenarios are used as input to the controller. The drive cycle block defines the desired forward velocity of the vehicle based on a particular driving scenario. These driving scenarios vary from highway driving (Highway Fuel Efficiency Test — HWFET) and stop-and-go traffic (New York City Cycle — NYCC) to repetitive acceleration and deceleration cycles Fan (2007). A number of them are defined and provided by the United States Environmental Protection Agency (EPA). Furthermore, the drive cycle block describes the road profile that the vehicle passes over. The road profile defines the current grade of the road and it is determined independently from the desired speed. The grade is defined as a function of distance traveled whereas the speed profile assesses the desired forward velocity based on the current time. After taking the current grade into account and comparing the velocity set by the speed profile with the current speed of the vehicle, the power controller calculates the operation modes, the power demands and the required torque (whether it is negative for deceleration or positive for acceleration).

**Power Control Logic**

The purpose of the power controller is to determine the power or braking demand of the vehicle to ensure the vehicle follows the drive cycle. In addition, it also controls the operation of the tractive motor, the regenerative braking, and the genset control based on the desired speed given by the drive cycle. The power controller is further divided into three subsystem blocks: drive controller, motor/generator mode logic, and genset logic.

**Drive control subsystem:** The drive controller subsystem utilizes a model-based feed forward and a PID controller to determine the driving and the braking demand of the vehicle to follow the desired speed. The controller receives the desired speed, acceleration, and the road grade to calculate the required torque necessary to achieve the desired acceleration while overcoming the external loads on the vehicle. The PID controller then further reduces the velocity error by applying a corrective torque based
on the speed difference between the desired and the actual speed.

*Electric motor/generator mode subsystem:* The electric motor/generator logic subsystem determines the motor mode of the electric motor/generator unit. Depending on the current and the desired vehicle states it determines whether the motor/generator unit should drive the vehicle or generate power through the regenerative braking system.

*Genset logic subsystem:* The genset logic subsystem consists of a simple control logic, where it activates the engine and the generator to charge the battery when the current charge of the battery pack drops below a certain value, and stops charging once the battery reaches another predefined level of charge. Those values define the energy management strategy of the HEV.

**IC Engine/Generator Setup**

The purpose of the IC engine/generator assembly, is to generate electrical power to recharge the battery, and consists of the IC engine and the generator component. It is assumed that the output shaft of the IC engine and the input shaft of the generator are directly connected with no transmission in between.

*IC Engine:* The IC engine model is fully defined by test data in the form of characteristic motor curves, which define the motor torque based on the state variable of the drive shaft rotation and vice versa, and efficiency maps. The desired engine torque is received by the genset control subsystem from the power controller block, and is limited by the maximum output torque available. The motor performance history is also used to calculate the fuel consumption based on the predefined model data input.

*Generator:* The generator receives the torque and speed output of the IC engine, and calculates the generated electric power based on the maximum torque and the efficiency of the generator. It also receives the power demand from the power controller. When the power generated is more than the power demand of the electric motor, the remaining power is used to recharge the battery pack.

**Battery and Auxiliary Loads**

The battery is the primary electrical energy storage system of the vehicle, where it stores the electrical power generated by the IC engine/generator combination and the regenerative
braking, while providing power to the traction motor and auxiliary requirement of the vehicle. The battery receives the motor power demand, auxiliary power consumption, generator power, and the regenerative braking power as inputs. The amount of current from each input is then calculated separately, and summed to determine the net current going into or out of the battery system, constrained by the charge/discharge limit of the battery. The model is based on the description of the state of charge (SOC) of the battery, which is used to calculate the power drawn by the motor and auxiliary systems or provided from the IC engine/generator series setup. Also the SOC defines the battery limits that are defined by the recharge strategy that depends on the energy management.

**Electric Motor/Generator**

The purpose of the motor/generator unit is to provide the tractive effort of the vehicle during acceleration, while performing regenerative braking during deceleration. Therefore, the system is divided into a motor and regenerative braking subsystem of which only one is active at a time. Which of the two is active is defined by the mode sent from the motor/generator logic subsystem. The motor/generator system then calculates the torque that is sent to the transmission and applied to the wheels of the vehicle dynamics block.

**Motor Mode:** The motor mode subsystem receives the electric power output from the battery and converts it to the desired output torque. The desired output torque is then compared against the maximum torque output capability of the electric motor. The appropriate torque output is then subsequently used to drive the vehicle model while taking motor efficiency into consideration.

**Regenerator Mode:** The re-generation mode subsystem of the motor/generator block calculates the brake torque based on the regenerative braking capabilities of the generator. The torque output is limited by maximum value that the unit can produce in generator mode. The electrical power from regenerative braking is calculated to recharge the battery, while considering the efficiency of the unit.

**Transmission**

The purpose of the gear box unit is to transmit the motor torque and speed depending on the current gear ratio determined from the predefined shift logic, and to distribute the torque
according to the vehicle drive configuration, i.e. front-wheel drive, rear-wheel drive or four-wheel drive. This unit resembles a two shift transmission where the shift logic can be easily changed according to the energy management of the vehicle.

**Vehicle Dynamics**

The vehicle model consists of a vehicle body representing the curb mass and two Fiala tire models for the front and rear axle. This means that the tires of each axle are combined and modelled as one tire. A schematic of the vehicle model including the main geometry parameters and external forces is given in Figure 3.2. The main body of the vehicle is constrained to the pitch plane coordinates and the wheels are connected with revolute joints to the vehicle. Since only straight line motion is simulated, no steering is included in the model which results in a model with five degrees of freedom (DOF): translation of the body in x and z-directions and rotation of the vehicle body and each wheel around the y-axis, where the ISO coordinate system is considered as a global reference frame. The direction of the x and z-coordinate can be seen in Figure 3.2 and the y-direction is given by the right handed coordinate system convention. Furthermore, the tire slip is calculated using an extra DOF and an extra parameter, the relaxation length. Therefore, the dynamics model and the resulting system of equations consist of 14 state variables. The vehicle dynamics are symbolically computed using Maple11 and DynaFlexPro that are used later on in a Simulink S-function block.

![Figure 3.2: Schematic of vehicle dynamics model](image)

The schematic depicts the current road grade \( \alpha \) and the vehicle parameters. These
include the vehicle mass $M$ and pitch inertia $I$, tire mass $m_T$ and inertia $I_T$, and the major geometry parameters of the vehicle that include the height $h$, distance of the mass center from the rear wheel $a$ and the front wheel $b$. The sum of $a$ and $b$ describes the wheel base of the vehicle. Additionally, the transmission inertia $I_{\text{trans}}$ and motor inertia $I_{\text{mot}}$ are modelled in this vehicle dynamics block as additional inertia applied to the tire. These are added together by taking the appropriate gear ratio $\gamma$ into account.

The purpose of the vehicle dynamics block is to calculate the location and velocity of the vehicle at every time step using the external forces (shown in Figure 3.2) including the vehicle weight $W$, the aerodynamic drag $D$, the tire forces $F_x$ and $F_z$, and the rolling resistance torque $M_{RR}$. The dynamics block determines the path driven by the vehicle and its velocity profile.

### 3.1.2 Simulation and Results

As stated earlier, the presented model is developed to simulate different drive train configurations and evaluate their relative performance to each other based on efficiency of each single component and the overall configuration. For that purpose, several simulations have to be run when the component of interest is varied according to the driving scenario. These driving scenarios can vary from predefined velocity profiles simulating efficiency during highway or city driving to acceleration test, simulating performance of different drivetrain setups. To demonstrate the performance of the model, the vehicle is simulated to follow the commonly used highway fuel efficiency drive cycle (HWFET) and the results are presented in Figures 3.3 to 3.5.

Figure 3.3 shows the desired velocity profile in comparison with actual speed. Evidently, the vehicle follows the desired speed closely which can also be seen in the plot of the drive cycle velocity error in Figure 3.4.
For the drive cycle presented here, the peak error is smaller than 0.5% and it appears during the acceleration phase which is 320 s into the simulation as the vehicle stays slightly below the desired speed at all times.
To demonstrate the versatility of the model and the evaluation methods, Figure 3.5 illustrates the energy flow between the battery pack and the electric motor/generator and the energy consumption of the electric motor. In the power exchange plot it can be seen that the vehicle switches between motor mode and regenerative braking mode over the duration of the drive cycle. Negative values depict the usage of electrical energy whereas positive values imply that the generator feeds back energy to the battery pack. This also shows up in the energy consumption plot as the overall consumed energy decreases whenever the electric motor is in generator mode. This can be best seen at the end of the drive cycle during the largest braking phase when the vehicle decelerates down to 0 m/s. As mentioned earlier this is just one way of showing the performance of particular components of the vehicle drive train and this method can be used to evaluate many different performance parameters of single components and the overall system.
3.2 Off-Road Tire/Soil Interaction Model

An off-road tire model for use in multibody models is developed including both a tire model and a soft soil model. The tire model is based on a radial-spring tire model (Davis 1974), whereas the soil model is based on a hyperelastic material model according to Bekker (Bekker 1969) (Bekker 1962). These models are then combined to calculate the normal force on the tire $F_z$, which supports the vehicle dynamics in vertical direction and is used to determine the remaining forces. An exponential model is used to calculate the traction force $F_x$ and the lateral force $F_y$ that are based on the kinematic tire properties of longitudinal slip $S$ and slip angle $\alpha$ (Metz 1992). To check the consistency of the tire model, it is simulated using a bicycle vehicle model which is a simplified model often used in vehicle dynamics simulations.

To calculate the forces due to the interaction of the tire with the soft soil, a conventional tire model is used and extended by a hyperelastic soil model as it can be seen in the simplified schematic in Figure 3.6. The tire forces are calculated using a radial tire model in which the representative tire deflection or in this case the deflection of the radial springs is based on the penetration area of the contact problem. The contact area between the tire and the soil is constantly changing given that as the tire rolls over the soft soil foundation, it sinks into it. The penetration area can be calculated by knowing the maximum sinkage of the tire into the soil which is then compared with a well understood tire model to calculate the magnitude of the tire force. Knowing the magnitude of the tire/soil contact force in normal

![Figure 3.6: Schematic of off-road tire model in contact with soil model](image-url)
Progress to Date

direction, the tire loads $F_x$ and $F_y$ can be calculated. For these calculations, it is necessary to combine the tire model with the soft soil model.

3.2.1 Calculation of Vertical Tire Force

Due to the compliance of the soil, the tire sinks into the ground which changes the contact patch geometry and pressure distribution as shown in Figure 3.7. These changes highly depend on the dynamics of the soft soil (Schmid 1995), which in this case is based on a Bekker’s soil model and explained in the following section.

The pressure distribution results in local deformation of the tire in the area of the tire patch which results in a reaction force and is used to calculate the tire forces. The deformation of the tire can be related to the penetration area. In order to simplify the force calculation, the penetration area is compared to the penetration area of a tire on a flat surface.

Calculation of the penetration area

To calculate the penetration area, the shape of the ground profile has to be defined. The soft soil geometry can be found by calculating the tire sinkage and assuming a basic shape of the profile. This can be determined by calculating the equilibrium between the vertical tire force
and the soil compression force which is based on Bekker’s hyperelastic soil model:

\[ p = k_0 z^n \quad \text{with} \quad k_0 = \frac{k_c}{b} + k_\phi \] (3.1)

where \( p \) is the pressure in the contact patch, \( b \) is the tire width, \( k_c, k_\phi, n \) are the Bekker soil parameters, and \( z \) is the compression of the soil. Since the Bekker parameters are obtained from a soil compression test, one has to distinguish between the soil deflection \( z \) and the maximum tire sinkage into the soil \( z_{\text{max}} \). However, the two variables can be related by integrating Bekker’s pressure distribution over the tire contact patch. This relation depends on the contact modelling approach and the assumptions made for the contact geometry. For the model described in this chapter, the assumption that the ground profile adapts a parabolic shape (see Figure 3.8) is made, which allows to fully determine the geometry of the contact problem. Taking the undeformed geometry of the wheel into account, the tire penetrates the ground as illustrated in Figure 3.8(a). Since both the geometry of the tire and the ground profile are known, it is a matter of simple geometry to determine the penetration area, \( A_{\text{comp}} \) — the intersection of the two geometries.

![Figure 3.8](a) Soil profile    (b) Comparison profile

Figure 3.8: Comparison of penetration areas for force calculation

To calculate the magnitude of the contact force, the actual penetration area as shown in Figure 3.8(a) is used as the penetration area \( A \) of a tire on a rigid road as shown in Figure 3.8(b). This is based on the assumption that the penetration area on a soil profile and a rigid road are the same (Davis 1974). Thus, the tire deflection of the simplified case can be determined and the magnitude of the resultant force is calculated assuming the following
equation:

\[ F_{\text{Tire}} = k_T \cdot f_T - c_T \cdot \dot{z}_{max} \]  

(3.2)

where \( k_T \) and \( c_T \) are the tire stiffness and damping respectively and \( f_T \) is the tire deflection of the simplified case. For simplification, \( \dot{z}_{max} \), the tire sinkage velocity into the ground, is used for the calculation of the damping term. The resultant tire force is the equivalent tire force \( F_{\text{Tire}} \), which represents the tire/soil interaction force \( F_{\text{soil}} \) of the original problem. The location of action of the contact force \( F_{\text{soil}} \) is assumed to coincide with the centroid of the actual penetration area as illustrated in Figure 3.9. From this representation of the contact force, the tire forces due to the interaction with the soil ground are calculated with the assumption that the reaction force from the soil acts at the centroid of the described penetration area normal to the circular shape of the wheel. This assumption of location and direction of the reaction force leads to two components: a force in \( z \)-direction representing the vertical tire force and a force in \( x \)-direction opposing the driving direction. The former represents the force component that supports the wheel load \( F_Z \), whereas the latter is an additional resistance to the forward motion of the vehicle and it represents the resistance that the vehicle has to overcome due to the tire sinkage into the soft soil. It can be understood as the losses due to the compaction of the soil.

### 3.2.2 Calculation of Longitudinal Tire Force

As the vertical force, the horizontal or longitudinal force that defines the tractive effort of a vehicle highly depends on the soil mechanics. Therefore, Janosi et al. suggested the following
exponential model to describe the shear stress-strain curve of soil (Janosi and Hanamoto 1961):

\[ \tau = \tau_{\text{max}} \left( 1 - e^{-\frac{j}{K}} \right) \] (3.3)

where \( K \) is the soil deformation modulus, \( j \) is the shear deformation and \( \tau_{\text{max}} \) is the maximum shear stress that can be calculated using the Mohr-Coulomb failure criteria:

\[ \tau_{\text{max}} = c + p \tan \phi \] (3.4)

where \( c \) is the effective cohesion, \( \phi \) is the angle of friction and \( p \) is the normal pressure. With the assumption of uniform stress, the soil shear deformation can be described as a function of the location of the soil particle on the tire/soil interface, where the longitudinal slip is the proportionality factor. The traction force can then be obtained by integrating the empirical soil model of the shear relation over the tire/soil interface.

\[ F_x = A \tau_{\text{max}} \left( 1 - \frac{K}{S L} e^{-\frac{S L}{K}} \right) \] (3.5)

where \( A \) is the contact patch area, \( L \) the length of the tire/soil interface and \( S \) the longitudinal slip which is calculated using the Equation 3.6. The derivation of Equation 3.5 can be seen in Appendix A.

\[ S = 1 - \frac{v_x}{R \omega_y} \] (3.6)

With the assumption of no cohesion, which is valid for dry and sandy ground (Harnisch 2005), the traction force model can be further simplified:

\[ F_x = \tan \phi \left( 1 - \frac{K}{S L} e^{-\frac{S L}{K}} \right) F_z \] (3.7)

where \( F_z \) is the vertical tire force as previously calculated, \( v_x \) is the longitudinal component of the wheel velocity, \( R \) the undeformed tire radius and \( \omega_y \) is the wheel spin. To avoid numerical problems, the longitudinal force is set to zero when there is no wheel spin \( \omega_y = 0 \). The resistance due to the soil compaction is subtracted form the traction force to obtain the longitudinal tire force, which is then applied to the vehicle model.

### 3.2.3 Calculation of Lateral Tire Force

The same exponential model for the shear stress and shear displacement relation based on the Mohr-Coulomb equation is used to model the lateral forces of the tire running on soft soil.
With another assumption that the soil shear displacement in the contact patch is constant over the tire width in lateral direction, the following equation for the lateral force can be found Metz (1992):

\[
F_y = A_{\text{lat}} \left( 1 - e^{-B_{\text{lat}} \alpha} \right) \tag{3.8}
\]

with \( \alpha = \arctan \frac{v_y}{v_x} \tag{3.9} \)

where \( A_{\text{lat}} \) and \( B_{\text{lat}} \) are the lateral parameters associated with the tire and the terrain respectively, \( \alpha \) is the lateral slip angle and \( v_y \) is the lateral component of the wheel velocity. Besides the fact that the model is analogous to soil shear phenomena as it is based on a curve fit of soil shear stress-strain tests, the model includes the effect of the vertical load on cornering. However, no camber effect is included.

### 3.2.4 Rigid Wheel

For comparison, a second tire model was created assuming a rigid wheel. This approach changes the calculation of the vertical tire force, whereas the models for the forces in longitudinal and lateral direction are identical. However, these forces are dependent on the vertical tire dynamics and are influenced by rigid wheel properties. The assumption of rigidity is considered valid for cases where the soil is relatively soft in comparison with the vehicle tires (e.g. dry sand). In fact, for many planetary rover designs with metal tires, the rigid wheel model is the better choice (Harnisch 2005). A schematic of the rigid wheel in contact with soft soil is shown in Figure 3.10. Assuming that the vertical pressure under the wheel may be expressed by Bekker’s soil compression model, the vertical tire force can simply be calculated with the equilibrium of the vertical forces and by integrating the Bekker stress distribution in \( x \) direction (Bekker 1962):

\[
F_z = \int_{0}^{z_{\text{max}}} b \left( \frac{k_c}{b} + k_\phi \right) z^n dx
\]

with \( p(x) = \left( \frac{k_c}{b} + k_\phi \right) z^n \)

where \( b \) is the width of the tire and \( p(x) \) is the Bekker stress distribution including the corresponding Bekker soil parameters. From the geometrical configuration shown in Figure
Figure 3.10: Schematic of rigid wheel in contact with soft soil

3.10 the following relations are used to solve the integral shown in Equation 3.10.

\[
\overline{AB} = \left(\frac{D}{2}\right) - (z_{\text{max}} - z) \quad (3.11)
\]

\[
x^2 = \left(\frac{D}{2}\right)^2 - (\overline{AB})^2 \quad (3.12)
\]

Substituting Equation 3.11 into Equation 3.12 results in the following relation:

\[
x^2 = [D - (z_{\text{max}} - z)] - (z_{\text{max}} - z), \quad (3.13)
\]

which can be approximated by,

\[
x^2 = D(z_{\text{max}} - z) \quad (3.14)
\]

Finally, the following Equation 3.15 can be deduced from Figure 3.10.

\[
2x \, dx = -D \, dz \quad (3.15)
\]

Substituting Equation 3.14 and 3.15 into Equation 3.10 leads to the following integral that needs to be solved for the vertical tire reaction:

\[
F_z = b \left(\frac{k_c}{b} + k_\phi\right) \int_{0}^{z_{\text{max}}} z^n \frac{\sqrt{D}}{2\sqrt{z_{\text{max}} - z}} \, dz \quad (3.16)
\]
which results in the following resultant tire force in vertical direction.

\[ F_z = 3 - \frac{n}{3} \sqrt{D} z_{max} b \left( \frac{k_c}{b} + k_\phi \right) z_{max}^n \quad (3.17) \]

With the geometry of the rigid wheel and the vertical force equilibrium, the maximum sinkage for the wheel into the ground can be calculated as follows (Janosi 1961):

\[ z_{max} = \left[ \frac{3 F_z}{(3 - n)(\frac{k_c}{b} + k_\phi)b \sqrt{D}} \right]^{\frac{2}{n+1}} \quad (3.18) \]

Since this calculation leads to a vertical force only, the energy losses due to the compaction of the soft soil can not simply be extracted from the normal force calculation of this tire model approach. This tire model and the previously suggested approach using a deformable tire are simulated and compared with each other and the results are presented in the following section.

### 3.2.5 Simulation and Results

In order to evaluate the performance of the tire model in interaction with the soft soil, a simple vehicle model was created consisting of a chassis, two steering knuckles and four tires. The center of mass of the vehicle is horizontally aligned with the wheel hubs and the tire radius is set to 0.59 m and the soil parameters are kept constant throughout every simulation to represent a sandy loam type of ground. The parameters and key values for the simulation are presented in Appendix B.

Three different scenarios are analyzed to compare the deformable tire model with the model using the assumption of a rigid wheel. The first simulated scenario is the settling phase in which the vehicle is driven at a constant speed until it settles in the vertical direction. Moreover, the tire stiffness in the deformable tire model is increased incrementally in order to investigate whether it approaches the same results for the settling analysis as the infinitely stiff tire model. The second analysis is an acceleration manoeuvre for which the vehicle undergoes a predefined acceleration cycle. The third and last scenario is a lane change manoeuvre in which the vehicle performs a sequel of a left and a right turn. In all simulations, the vehicle is driven by motion drivers applied to all four wheels.

The results of all simulation runs are presented in the plots of Figures 3.11 to 3.13.
Figure 3.11 shows the vertical position of the center of mass (CoM) location in vertical direction of the main body. The plot shows the comparison between the deformable and the rigid tire model. To compare the compatibility between the two models, two more simulations are performed where the stiffness of the deformable tire is increased.

Comparing the deformable tire with the rigid tire model rolling on soft soil it can be seen that the CoM of the vehicle settles approximately 3 cm lower in case of the deformable tire model. This small difference represents the tire deflection in vertical direction that is missing in the model of the rigid tire; however, it is taken into account for the deformable tire model. This can also be seen in the results of the incrementally stiffened tire models. The stiffer the tire becomes, the smaller the difference between it and the rigid model and all that remains is the soil deflection. It can also be seen that the vehicle settles after approximately 1.5 s. Therefore, in the following simulation, the initial conditions are set closer to the CoM height of the vehicle after settling and the vehicle is given 2 s to settle before it performs the defined manoeuvres.

Figure 3.12 illustrates the performance during acceleration of the simulated tire models in comparison with each other where the forward velocity is plotted over the time during an acceleration manoeuvre. In both cases vehicle starts with a constant velocity and then accelerates for three seconds before it settles back into a higher constant velocity state.
accelerate the vehicle all four wheels are driven through a motion driver in the revolute joint of each wheel, which represent the wheel hub. The acceleration profile is applied after a short settling period.

In the results of the acceleration test, it can be seen that the vehicle with the rigid tire model settles to a slightly lower velocity since the resistance due to soil compaction is not simulated in the rigid tire model. Furthermore, the rigid tire reacts faster to the acceleration input and it also accelerates to a higher final speed during the same acceleration sequence. This means that the losses are less compared to the deformable tire, which is the obvious result of the compression losses simulated in the deformable tire but not in the rigid tire. Also, the lesser dissipation in the rigid tire is a result of the vertical dynamics and the kinematics of the tire model. The longitudinal force is directly dependent on the vertical force, which is commonly overestimated for rigid tire properties. Since the parameters of the longitudinal force model are the same for both models, the rigid tire model calculates higher longitudinal force simply because of the higher vertical force. In addition, the kinematics of the tire and especially the longitudinal slip change due to the difference in velocity, which is also a reason for the differing results.

The results of the lane change manoeuvre shown in Figure 3.13 depict the performance of the lateral force description of the tire models. The vehicle runs through a slight left and
right turn which is defined by a steering function in the form of a sine curve with a period of 10 s and a magnitude of 2°. Once again when the rigid wheel model is compared against the deformable tire, it can be seen that the vehicle with the rigid wheel model is again more efficient with respect to the steering input. This is due to the same reason discussed earlier for the longitudinal force model. The lateral force is directly dependent to the vertical tire force, and since the rigid tire properties result in higher normal contact forces, the lateral forces are higher than the forces calculated using the deformable tire. The vehicle with the rigid wheels appears to have a delayed response to the steering input; however, this is simply the result of a higher velocity of the vehicle with the rigid wheel. Again, the difference in the tire kinematics also influences the results since the lateral force model is a function of the slip angle. Furthermore, the higher lateral forces of the rigid wheel model result in more lateral slipping which is due to the higher acceleration in lateral direction. This can be observed from the heading direction of each model after the lane change manoeuvre. The vehicle with the rigid tires is deflected more from the x-direction than the vehicle using the deformable tire model.
Future Work

The main objective of this project is to develop math-based models of physical systems and subsystems related to electric vehicle designs and planetary rovers in particular. This objective will be achieved by improving the current modelling and design techniques of electric vehicle components and off-road tires in contact with soft soils. Furthermore, these models will be validated with experimental results provided by the CSA and by comparing the model with numerical models created in different software packages.

4.1 Math-Based System Modelling

The primary goal is to develop formulations that can automatically generate the equations of motion for systems and subsystems of electric vehicles with a particular focus on planetary rovers. For the electric vehicle and planetary rover application, it is necessary for the models to include electrical, mechanical, electro-mechanical and drive train components (e.g. battery, transmission and electric motor/generator). Other models will focus on the contact modelling between mechanical components such as brakes and tires. The overall process involves two distinct phases: the modelling phase and the simulation phase.

The modelling phase will focus on generating the equations of motion for a given set of generalized coordinates, generalized velocities and system parameters. These equations will be used to simulate the system dynamics for different scenarios, which is referred to as the simulation phase. A block of a system used for forward dynamic simulation can be seen in Figure 4.1. The model calculates the generalized accelerations $\ddot{p}$ and the reactions enforcing
the constraints of the system $f$. These quantities are calculated using the generalized coordinates $q$ and velocities $p$ and the system parameters which define the constitutive equations of the components and the geometry of the system (Samin and Fisette 2003). By defining the simulation time and the initial conditions of the system states, the system can then be simulated which is a numeric process.

![Block diagram of forward dynamic analysis](image)

Figure 4.1: Block diagram of forward dynamic analysis

For this project, the focus will be on the modelling phase. This phase differs from one method to another depending on the formulation being used. The formulation defines the sequel of essential steps or procedures that need to be performed to generate the equations of motion in multibody dynamics. In general, these steps are based on fundamental principles and the formulations can be of numerical or symbolic nature. In the modelling phase, the formulation generates the dynamics of a system given the constitutive equations of each of its components and their interconnections — the topology of the system.

Numerical formulations build the system equations by creating structures of numerical matrices at a certain instant in time. This means that the model has to be rebuilt in every time step which is a slow process as it involves calling several subroutines that are required for the formulation of the system. The most known software package using numerical formulations to generate the equations of motion of multibody systems is the simulation tool MSC.ADAMS.

In contrast, symbolic formulations create a set of differential-algebraic equations (DAEs) that define the system dynamics for all times. For a given multibody system, this symbolic generation is only performed once. This is the main reason for the computational advantage of a symbolic formulation. In addition, terms multiplied by 0 are automatically removed by Maple (unlike the numerical approach), and various simplifications and reductions can be applied symbolically during the formulation of equations. All this leads to a significant time advantage in simulating symbolically generated equation over those generated using the numerical formulations. Besides the advantage of fast analysis, symbolic models are very flexible and can be easily exported to any simulation language. Therefore, the formulations will be developed in a symbolic programming language, so that the final equations can be
easily viewed and shared among others, who can change system parameters to create their own math-based models for various simulations.

Linear graph theory will be used as a tool for the development of the symbolic formulations that can sufficiently generate models of not only electric vehicle systems and subsystems, but also contact between two compliant bodies, e.g. tires in contact with soft soil. In linear graph theory, the system is modelled as a collection of system elements (“edges”) and their interconnection (“nodes”). The edges represent the components of the system with their constitutive model, whereas the nodes define the topology of the system.

The system equations are directly obtained from the linear graph representation, in combination with the characteristic equations for each component (e.g. \( F = ma \) for a rigid body). In particular, the graph-theoretic Principle of Orthogonality is used to automatically formulate the equations for the multi-domain system (Schmitke and McPhee 2007):

\[
\sum_{i=1}^{n} x_i \cdot \delta y_i = 0 \quad (4.1)
\]

\[
\sum_{i=1}^{n} y_i \cdot \delta x_i = 0 \quad (4.2)
\]

where \( x_i \) and \( y_i \) are the “across” and “through” variables, respectively, assigned to each of the \( n \) elements in the graph. A through variable is measured by an instrument in series with the physical element, e.g. current in the electrical domain and force for mechanical domain. An across variable is measured by an instrument in parallel with the element, e.g. electrical voltage and mechanical position, velocity, or acceleration. The \( \delta \) operator corresponds to a virtual change in the accompanying variable, \( x \) or \( y \). Thus, equation 4.1 is the projection of the across variables onto the “through space” of the system; for mechanical systems, this gives the algebraic constraint equations between the system coordinates. Equation 4.2 is the projection of the through variables onto the “across space” of the system; for mechanical systems, this gives the dynamic equations of motion. If the Principle of Orthogonality is written without the \( \delta \) operator, a general energy conservation principle is obtained (Andrews and Kesavan 1973a). When equations (4.1 - 4.2) are evaluated for mechatronic systems, a set of equations in the following matrix form are obtained:

\[
M \dot{p} + C^Tf = b(p, q, t) \quad (4.3)
\]

\[
\Phi(q, t) = 0 \quad (4.4)
\]

\[
\dot{q} = h(p, q, t) \quad (4.5)
\]
where \( \mathbf{p} \) and \( \mathbf{q} \) are vectors of the generalized speeds and coordinates, respectively. For mechanical systems, \( \mathbf{M} \) is a mass matrix. For electrical systems, the corresponding entries in \( \mathbf{M} \) will be inductance terms. The column matrix \( \mathbf{f} \) contains variables that enforce the nonlinear algebraic constraint equations in the column matrix \( \Phi \), and \( \mathbf{C} \) is the corresponding coefficient matrix. Equation 4.5 represents the transformation between generalized speeds and the derivatives of generalized coordinates; on the right-hand side \( \mathbf{b} \) and \( \mathbf{h} \) are both nonlinear functions of \( \mathbf{p} \), \( \mathbf{q} \), and time \( t \). Equations (4.3 - 4.5) constitute a set of differential-algebraic equations (DAEs) that can be solved numerically.

### 4.2 Math Based EV Modelling

In the past, linear graphs provided an effective way to model 3D mechanical systems as well as mechatronic systems. Now they will be used to develop formulations to symbolically model electric vehicle systems and subsystems. The subsystems of an electric vehicle include each component of the empirical model (described in Section 3.1), which are the electric motors and generators, storage systems (e.g. battery), transmission, and the vehicle dynamics.

The models of these electric vehicle components will be used as the basis for an electric vehicle design. The graph-theoretic formulation will have all the domains necessary to develop EV models and will be used as a tool to do so in the symbolic modelling environment given by Maple and its simulation toolbox MapleSim. Electrical as well as mechanical models will be needed for the generator, battery, motor and transmission. Electrical motors have already been modelled in the past, and a generator is often modelled as a motor operating in reverse. For battery models though, where state of charge (SOC) is used as a state variable, the highly nonlinear components are often modelled using numerical data sheets in tabular form. This procedure has been widely used in the past, especially for models of Lithium-Ion batteries which are most commonly used in high performance EV applications Nelson et al. (2002a). A schematic of the equivalent circuit of a Li-Ion battery model can be seen in Figure 4.2.
In Figure 4.2, $OCV$ is the open circuit voltage, $C_0$ and $R_0$ are the internal capacitance and resistance respectively, $C_p$ and $R_p$ are the polarization capacitance and resistance respectively, $V_L$ is the battery terminal voltage and $I_L$ is the discharge current that defines the load on the battery. The lumped parameter model of the battery in Figure 4.2 can be easily represented using linear graph theory where the constitutive equation of each component is of nonlinear nature. Examples of nonlinear electric components and their constitutive models can be reviewed in (Chandrashekar and Savage 1997). Also, the components can be represented using experimental data. The $OCV$, which can be easily measured, is often used to determine the battery performance. Plotted over the $SOC$ of the battery, it defines the characteristic curve of the battery and can be used to model the constitutive equation of the $OCV$ component shown in Figure 4.2. A symbolic computer implementation of such models will require the conversion of tabulated data to functional form. It is expected that piecewise cubic splines will be suitable for this step, although other interpolation functions will be investigated.

Graph theory will also be used to integrate the many components into a single system model, and past work on subsystem modelling as well as the previously shown empirical HEV subsystem models (see Section 3.1) can be used for validation purposes. The EV system model will be combined with a power controller that controls the flow of energy within the system.
4.3 Math Based Off-Road Tire Modelling

For planetary rovers, which are a specific type of electric vehicle, the dynamics are highly dependent on the interaction between the off-road tires and the environment, which can be soft soil or uneven terrain. The modelling of such a contact dynamic problem with friction is a very active research area; however most math-based contact models are simple in form and of low fidelity which leads to problems with multiple contact points and contacts with sharp edges. A contact modelling approach that has been sufficient for such contacts will be applied to this particular application. The approach uses volumetric properties of the penetration geometry to calculate the normal forces. Depending on the complexity of the assumed tire geometry, this approach is assumed to allow for symbolic high-fidelity modelling of contacts. The expected complexity of such models will also create significant challenges for a symbolic computer implementation as the calculation of the volume metrics require high computational effort. To overcome this obstacle, the project partners and 3D graphical computing specialists from Parallel Geometry Inc. (LLG) will provide the hardware and software required for the geometric modelling. Their high-performance algorithms installed on appropriate computation hardware will be exploited to efficiently obtain the volume metrics. Thereby, another challenge will be to combine the provided software with Maple.

The normal force calculation for the any symbolic contact model is a crucial step and it will define the overall performance of such a model. Symbolic tire and off-road tire models are no exception. All six force and moment components will be defined by this normal pressure distribution as was shown in the simplified off-road tire model in Section 3.2. In this project, the volumetric contact will serve as the fundamental theory for this normal force calculation of the off-road tire in contact with soft soil. The compliant ground will be represented by the Bekker’s soil model. Since both of the two contacting bodies in the tire/soil interaction model are considered compliant, one of the challenges of this project is to determine the current deformed geometries of the objects. Due to the highly compliant properties of the soil in particular, the deformations of the two bodies have to be determined before the interpenetration volume can be calculated based on the current location and the deformed states of each object.

The volumetric contact model will serve as the basic building block for the modelling of the off-road tire model. However, the formulation of the volumetric contact model must be modified to account for different types of normal and shearing loads when modelling the interaction of a wheel on the ground, i.e. soft-soft contacts. The volumetric contact model
Figure 4.3: Torus in contact with ground

is based on a Winkler elastic foundation model that predicts the local contact pressure as a function of the local penetration depth. The effects of local contact pressure is summed over the entire contact area to obtain the overall normal force $F_V$ as a function of the volumetric stiffness $k_v$ and damping $c_v$, the volume of interpenetration $V$, and the relative velocity between the two objects $v_{rel}$ (Gonthier 2007), as shown in Figure 4.3 and Equation 4.6.

$$F_V = k_v V (1 + c_v v_{rel}) \quad (4.6)$$

The elastic foundation for the tire will be replaced by the soft soil, modelled using the Bekker model. This is an empirical model that predicts the average contact pressure as a function of the width of the contact area, the soil properties and the maximum penetration depth (Bekker 1969). The Bekker model was used to analyze the dynamics of the off-road vehicle model in Section 3.2. The contact force found using the Bekker model was obtained by evaluating the penetration area based on the equilibrium between the vertical tire force and the soil reaction.

As a first step the tire geometry is considered to be undeformed when it is in contact with the soft soil, which does not mean that the tire possesses rigid material properties. It merely simplifies the calculation of the interpenetration volume, and for many off-road tire applications depending on inflation pressure and tire stiffness this is a valid assumption (Bauer et al. 2005b). Initially, low fidelity tires will be modelled using simplified tire geometries (e.g. cylinder, torus). These will be implemented on a commercially available desktop computer. Then higher fidelity models with more complex geometries, possibly obtained from CAD files, will be implemented on the high-performance hardware and software provided by
LLG. LLG has also made significant advances recently in 3D terrain modelling (Rotge and Farret 2007).

The intersecting volume is a function of the shape and size of the contact area as well as the penetration depth, i.e. $V$ is a function of the tire sinkage $z$ and and the contact area $A_c$. The goal here is to re-formulate the volumetric model such that it provides results that are consistent with the empirical Bekker model. Further methods to determine the volumetric stiffness parameter $k_v$ will be investigated, to avoid converting to an equivalent tire deflection to compute the normal tire load. The tire rolling resistance and self-aligning moments are naturally computed by the volumetric model (Gonthier et al. 2005). There is a tire model that calculates the penetration volume implemented in MSC.ADAMS. However, unlike this ADAMS 3D tire model and the planar off-road tire model shown in Chapter 3 in which the penetration properties are used to calculate a equivalent tire deflections, the volumetric contact model directly calculates the normal contact force as a function of the volumetric properties.

The lateral and longitudinal forces are calculated as a function of the normal force. Thus, once the normal force is calculated, existing models for the lateral and longitudinal tire forces can be implemented. Along with the overturning, the rolling resistance, and the self-aligning moments, one can calculate all six tire force and moment components according to the ISO coordinate system using Equation 2.2. This completes the tire model. The tire forces of the off-road tire model can be calculated and tested with respect to the effects of the tire kinematics and orientation (e.g. longitudinal slip, slip angle, camber angle etc).

The new off-road tire model will be combined with electric vehicle models to obtain dynamic models and simulations of planetary rovers that receive their power from solar panels. For simplification, the solar cells will be represented as energy source models and will be provided by the CSA. The end result will be a simulation facility for rovers moving on planetary terrains.

4.4 Validation

The new simulation infrastructure will make it possible to study and compare the performance of various rover concepts in a planetary context (Mars, Moon), as well as provide the possibility to operate a substitute virtual rover when the weather conditions do not permit operations with the real hardware.
4.4.1 Validation by Comparing with Other Approaches

The models that will be included in the planetary modelling and simulation facility will be compared to simulation results found from different simulation software packages and to simulations of similar models that have proved successful in the past. To validate the overall vehicle dynamics performance, the deployed rover design can also be modelled in other multibody software packages such as MSC.ADAMS, which generates the system equations numerically. To validate the models of the electric drive train subsystems, the developed models can be compared against the previously developed Matlab models described in Chapter 3. The off-road tire formulation can also be compared against models that were developed using different approaches such as ©AS2TM developed by AESCO Corporation. The ©AS2TM has been used by Bauer et al. (2005a) for planetary rover simulations.

4.4.2 Experimental Validation

To complete the validation process, all of the simulations will be compared against experimental results that will be provided by the CSA. The simulation results will be experimentally validated using the Planetary Exploration Test Facility (PlanET-F) mobile platform on the CSA Mars Emulation Terrain (MET), which is located on the grounds of the CSA in St. Hubert, Quebec. The test facility includes a fleet of robotic vehicles that can be tested. Most recently, the rover “Red” has been used for tests at the MET while being remotely controlled by an astronaut orbiting the Earth in the International Space Station (ISS). “Red” is equipped with lasers and it will transmit a 360 degree scan of its surrounding to provide 3D images. These images will be used for the input into the high-performance algorithm of LLG. Furthermore, the dynamics of the rover will be recorded during these tests. CSA will carry out the PlanET-F experiments and provide measurements that will be used to develop and validate the new models and simulations.
Summary

The significant addition to current simulation and design tools and a proposed outline with the expected milestones are listed in sections 5.1 and 5.2.

5.1 Contributions

The expected contributions from this work are as follows:

Development of a formulation that automatically generates the governing equations of planetary rover systems and subsystems using linear graph theory and symbolic computation. These subsystem models may include models of electric vehicle drive train components, e.g. electric motors, batteries etc., and also an off-road tire model based on a volumetric contact and Bekker’s soft soil model.

Implementation of the developed models into symbolic programming language Maple and LLG’s high-performance graphics software.

Application of the developed formulation to full vehicle modelling of planetary rover and simulation for various scenarios.

Validation of math-based models of off-road tire and electric drive train subsystem by comparison with previously developed models and implementations in other software package such as MSC.ADAMS. Moreover, employ experimental data obtained at the CSA planetary rover test facilities to compare against the simulation results.
5.2 Schedule

The proposed research schedule including the work done to date can be seen in Table 5.1.

<table>
<thead>
<tr>
<th></th>
<th>Course work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter 2008</td>
<td>SYDE 652: Dynamics of Multibody Systems</td>
</tr>
<tr>
<td></td>
<td>→ Developed full vehicle model for simulations of</td>
</tr>
<tr>
<td></td>
<td>hybrid electric drive train efficiency</td>
</tr>
<tr>
<td>Spring 2008</td>
<td>Research Assistantship</td>
</tr>
<tr>
<td></td>
<td>Development of HEV models for Magna Advanced Technologies</td>
</tr>
<tr>
<td></td>
<td>Course work</td>
</tr>
<tr>
<td></td>
<td>ME 610: Analytical Vibrations</td>
</tr>
<tr>
<td></td>
<td>→ Project: “Comparison of Golf Ball/Driver Impact</td>
</tr>
<tr>
<td></td>
<td>Models with Respect to Mechanical Impedance Matching”</td>
</tr>
<tr>
<td></td>
<td>Submitted to ASME Smart Material, Adaptive Structures and Intelligent Systems Conference 2009</td>
</tr>
<tr>
<td>Fall 2008</td>
<td>Course work</td>
</tr>
<tr>
<td></td>
<td>ME 620: Mechanics of Continua</td>
</tr>
<tr>
<td></td>
<td>→ Project: “Off-Road Tire Model for Contacts with Soft Soil”</td>
</tr>
<tr>
<td>Winter 2009</td>
<td>Further development of the off-road tire/soil interaction model, literature review and preparation for comprehensive exam.</td>
</tr>
<tr>
<td>Spring 2009</td>
<td>Develop new models for off-road tires based on Bekker soft soil model, by extending volumetric-type contact and using simple tire geometries, e.g. cylinder, torus.</td>
</tr>
<tr>
<td>Fall 2009</td>
<td>Attend training at LLG for high-performance graphics software. Purchase and install high-performance computer system, plus associated software. Develop volumetric contact models of off-road tire for complex tire geometries, and implemented in LLG hardware and software.</td>
</tr>
<tr>
<td>Winter 2010</td>
<td>Further develop these off-road tire models for complex geometries, and development of full planetary rover model.</td>
</tr>
<tr>
<td>Semester</td>
<td>Task</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Spring 2010</td>
<td>Further develop full planetary rover model including electric vehicle drive train subsystem models.</td>
</tr>
<tr>
<td>Fall 2010</td>
<td>Work with partners at CSA to obtain experimental data for their planetary rover prototype. Use to validate volumetric tire models and vehicle simulations. Begin preparation of thesis.</td>
</tr>
<tr>
<td>Spring 2011</td>
<td>Completion of thesis</td>
</tr>
</tbody>
</table>
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Appendix A

Integration of shear stress distribution

The total traction force is obtained by integrating the shear stress over the contact area. For simplification, the width $b$ of the tire is assumed to be constant. A schematic of the contact between the rigid wheel and the soft soil showing the shear deformation of a particle in the contact patch can be seen in Figure A.1.

\[ F_x = b \int_0^L \tau_{\text{max}} \left( 1 - e^{-\frac{j(x')}{K}} \right) dx' \quad \text{with} \quad \tau_{\text{max}} = (c + p \tan \phi) \quad \text{(A.1)} \]

where $c$ is the effective cohesion, $\phi$ is the angle of friction and $p$ is the normal pressure.
With the assumption of a uniform normal pressure distribution \( p(x) = \text{constant} \), the soil deformation \( j(x') \) under the the wheel is proportional to the location of the soil particle measured from the front end of the contact. The proportionality factor is the longitudinal slip \( S \). Then Equation A.1 can be written as:

\[
F_x = b (c + p \tan \phi) \int_{0}^{L} \left(1 - e^{-\frac{S x'}{K}}\right) dx' \quad \text{with} \quad j(x') = S x'
\]  
(A.2)

The integration yields to:

\[
F_x = b (c + p \tan \phi) \left[ L + \frac{K}{S} \left(e^{-\frac{S L}{K}} - 1\right)\right] \quad \text{(A.3)}
\]

For simplification, it is assumed that the resultant force equals the longitudinal tire force, although its direction does not always align with the \( x \)-axis. Moreover, using the following relations,

\[
A = bL \quad \text{and} \quad F_z = pA,
\]

Equation A.3 becomes:

\[
F_x = (Ac + F_z \tan \phi) \left[1 - \frac{K}{S L} \left(1 - e^{-\frac{S L}{K}}\right)\right] \quad \text{(A.4)}
\]

Finally, with the assumption of zero cohesion, the longitudinal tire forces are calculated using the following equation which can also be seen in Equation 3.7 in Chapter 3,

\[
F_x = \tan \phi \left(1 - \frac{K}{S L} e^{-\frac{S L}{K}}\right) F_z \quad \text{(A.5)}
\]

where the slip \( S \) is calculated using following expression:

\[
S = 1 - \frac{v_x}{R \omega_y} \quad \text{(A.6)}
\]
Appendix B

Parameters and Simulation Input

B.1 Tire and Soil Parameters

Tire Parameters (Harnisch 2005) (Uchida and McPhee 2007):

- Geometry parameters:
  - Unloaded Radius $R = 0.59 \, [m]$
  - Width $b = 0.32 \, [m]$

- Material parameters:
  - Stiffness $k_T = 375,000 \, [N/m]$
  - Damping $c_T = 4,500 \, [Ns/m]$

Soil Parameters:

- Normal force calculation (Bekker’s parameters) (Wong 1993):
  - Cohesive Modulus $k_c = 5,270 \, [N/m^{n+1}]$
  - Frictional Modulus $k_\phi = 1,515,040 \, [N/m^{n+2}]$
  - Sinkage Exponent $n = 0.7$

- Longitudinal force calculation (Harnisch 2005):
- Tangential modulus of horizontal shear deformation $K = 1.49 \text{ [m]}$
- Angle of Friction $\phi = 0.722 \text{ [rad]}$
- Cohesion $c = 0 \text{ [N/m}^2\text{]}$

- Lateral force calculation (Metz 1992):
  - Lateral max. load coefficient $A_{lat} = 0.88$
  - Lateral stiffness coefficient $B_{lat} = 0.85$

### B.2 Motion Drivers

Wheel spin function for acceleration manoeuvre:

$$\text{SPIN}(t) = \begin{cases} 
2\pi(t - 2)^2 + 4\pi t & \text{2} \leq t < \text{5} \\
16\pi(t - 5) + 38\pi & \text{5} \leq t \\
4\pi t & \text{otherwise}
\end{cases} \quad (B.1)$$

Steering input for lane change manoeuvre:

$$\text{STEER}(t) = \begin{cases} 
2^\circ \sin \left(\frac{\pi}{5}(t - 3)\right) & \text{3} \leq t < \text{13} \\
0 & \text{otherwise}
\end{cases} \quad (B.2)$$